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A NOTE ON NEW PARTIAL ORDERING OVER INTUITIONISTIC FUZZY SETS

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Abstract: A new partial ordering between intuitionistic fuzzy sets is proposed and investigated in the present paper. Some results concerning its relation to the classical ordering are given.
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1 Introduction

Intuitionistic fuzzy sets (IFS) were introduced by K. Atanassov (see [1]) as a generalization and extension of the concept of fuzzy sets. We will briefly remind some of the basic definitions and notions.

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Let X be a universe set, $A \subset X$, $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ are mappings reflecting the degree of membership and non-membership of the element $x \in X$ to the set A, respectively, such that for every x it is fulfilled that

$$\mu_A(x) + \nu_A(x) \leq 1 \tag{1}$$

Definition 1. Following [2], we call the set

$$A^* \stackrel{\text{def}}{=} \{x, \mu_A(x), \nu_A(x) | x \in E\}$$

an intuitionistic fuzzy set (IFS) and the mapping $\pi_A : X \to [0,1]$, which is given by

$$\pi_A(x) \stackrel{\text{def}}{=} 1 - \mu_A(x) - \nu_A(x), \tag{2}$$

is called **intuitionistic fuzzy index** (sometimes also: *hesitancy margin* or degree of *indeterminacy*) of the element x (cf. [6]).

Further we denote the class of all IFSs defined over a universe set X by $\text{IFS}(X)$.

Definition 2 (cf. [2, p.134, (7.1)],[6, p.43, Definition 3.4]). For a given IFS $A \in \text{IFS}(X)$ the *degree of definiteness* of the element x is said to be:

$$\sigma_{1,A}(x) \stackrel{\text{def}}{=} \mu_A(x) + \nu_A(x) \tag{3}$$

This degree provides an intuitive measure of the certainty of the knowledge established for the element. Indeed it is easy to see that it is directly related to *intuitionistic fuzzy index*, since for all $x \in X$, we have (due to (2)):

$$\sigma_{1,A}(x) = 1 - \pi_A(x).$$

Definition 3 (cf. [3]). An intuitionistic fuzzy pair (IFP) is an ordered couple of real non-negative numbers $\langle a_1, a_2 \rangle$, with the constraint:

$$a_1 + a_2 \leq 1. \tag{4}$$

This concept is very important in practice since many methods implementig IFSs, lead to estimates in the form of IFPs as is the case of InterCriteria Analysis [4]. One way to measure which result is "better" is by using some partial ordering on IFPs. The one most often used is the following

Definition 4 (cf. [3]). Given two IFPs $u = \langle u_1, u_2 \rangle$ and $v = \langle v_1, v_2 \rangle$, we say that u is less or equal to v, and we write:

$$u \leq v,$$

iff

$$\begin{cases} u_1 \leq v_1 \\ u_2 \geq v_2. \end{cases} \tag{5}$$

Remark 1. It is obvious that the above is a partial ordering, since it is transitive, reflexive and antisymmetric but there exist u and v, for which conditions (5) are not satisfied.

Remark 2. An equivalent form of (5) is:

$$\begin{cases} u_1 \leq v_1 \\ 1 - u_2 \leq 1 - v_2. \end{cases} \tag{6}$$

A formulation of the *necessary and sufficient condition* for the fulfillment of (5) is made obvious by (6) (due to (4)), namely:

$$\begin{cases} \min(u_1, v_1, 1 - u_2, 1 - v_2) = u_1 \\ \max(u_1, v_1, 1 - u_2, 1 - v_2) = 1 - v_2. \end{cases}$$

Another partial ordering between IFPs investigated by E. Marinov is the following:

Definition 5 (cf. [7]). Given two IFPs $u = \langle u_1, u_2 \rangle$ and $v = \langle v_1, v_2 \rangle$, we say that u is (π-based) less or equal to v, and we write:

$$u \preceq_\pi v,$$

iff

$$\begin{cases} u_1 \leq v_1 \\ u_2 \leq v_2. \end{cases} \tag{7}$$

Remark 3. We note that for any two IFPs u and v, at least one of the four is satisfied:

- $u \leq v$
- $v \leq u$
- $u \preceq_\pi v$
- $v \preceq_\pi u.$

This follows directly from the definitions but it is also evident that the two partial orderings are mutually exclusive except in the case of equalities.

Remark 4. The partial ordering $\leq$ over IFPs, defines inclusion over IFSs in the following manner. Let $A, B \in \text{IFS}(X)$. Then we say that A is included in B, and write $A \subseteq B$ if for all $x \in X$:

$$\langle \mu_A(x), \nu_A(x) \rangle \leq \langle \mu_B(x), \nu_B(x) \rangle \tag{8}$$

2 A new partial ordering between IFPs

In what follows we propose a new partial ordering between IFPs, and investigate its relation to the classical $\leq$ ordering from Definition 4. Recalling (3), let us first introduce

Definition 6. For a given IFP $a = \langle a_1, a_2 \rangle$ we will say that the *degree of definiteness* of a is:

$$\sigma(a) \stackrel{\text{def}}{=} a_1 + a_2 \tag{9}$$

Now we are ready to define the new partial ordering as:

Definition 7. Given two IFPs $u = \langle u_1, u_2 \rangle$ and $v = \langle v_1, v_2 \rangle$, we say that u is (definiteness-based) less or equal to v, and we write:

$$u \preceq_\sigma v,$$

iff

$$\begin{cases} \sigma(u)u_1 \leq \sigma(v)v_1 \\ \sigma(u)u_2 \geq \sigma(v)v_2. \end{cases} \tag{10}$$

The fact that $\preceq_\sigma$ is partial ordering (i.e. is transitive, reflexive and antisymmetric) is obvious. We will now establish some results concerning its relation with the $\leq$ ordering from Definition 4.

Theorem 1. If for two IFPs u and v, it is true that $u \preceq_\sigma v$, then it is also true that $u \leq v$.

Proof. Let us consider the only three possible cases:

i. $\sigma(u) = \sigma(v)$

ii. $\sigma(u) > \sigma(v)$

iii. $\sigma(u) < \sigma(v)$

If we have case i., then (10) is equivalent to (5). Hence, the assertion is correct.

Let us consider case ii. Then the first inequality of (10) yields:

$$u_1\sigma(v) \leq u_1\sigma(u) \leq v_1\sigma(v),$$

and hence we establish that $u_1 \leq v_1$. It remains to show that $u_2 \geq v_2$. But

$$\sigma(u) = u_1 + u_2 > v_2 + v_1 = \sigma(v),$$

and it follows that

$$u_2 > v_2 + \underbrace{(v_1 - u_1)}_{\geq 0},$$

Hence, $u_2 \geq v_2$, and this case yields again the validity of (5).

Lastly, let us consider case iii. Then the second inequality of (10) yields:

$$\sigma(v)u_2 \geq \sigma(u)u_2 \geq \sigma(v)v_2.$$

and hence we establish that $u_2 \geq v_2$. It remains to show that $u_1 \leq v_1$. But

$$\sigma(u) = u_1 + u_2 < v_2 + v_1 = \sigma(v),$$

and it follows that

$$u_1 < u_1 + \underbrace{(u_2 - v_2)}_{\geq 0} < v_1$$

Hence, $u_1 \leq v_1$, and this case yields again the validity of (5). This completes the proof. □

Remark 5. We have established that $\preceq_\sigma$ implies $\leq$. The converse is generally not true. For instance, let

$$u = \langle 0.3, 0.5 \rangle,\ v = \langle 0.42, 0.48 \rangle$$

We obviously have $u \leq v$, but $u \not\preceq_\sigma v$.

Definition 8. Let $A, B \in \text{IFS}(X)$. Then we say that A is included definiteness-wise in B, and write $A \subseteq_\sigma B$ if for all $x \in X$:

$$\langle \mu_A(x), \nu_A(x) \rangle \preceq_\sigma \langle \mu_B(x), \nu_B(x) \rangle \tag{11}$$

A direct Corollary from Theorem 1 is the following

Corollary 1. Let $A, B \in \text{IFS}(X)$. Then if $A \subseteq_\sigma B$, it follows that $A \subseteq B$.

3 Conclusion

In the present paper a new partial ordering between IFPs is introduced and its connection to the classical ordering is established. In passing a necessary and sufficient condition for the fulfillment of the classical ordering is given. Also, the proposed ordering introduces a new inclusion between intuitionistic fuzzy sets.

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