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A NEW OPERATION OVER INDEX MATRICES

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Abstract: A new operation over index matrices is introduced. Its properties are discussed and its relation w ith already defined operation is studied.
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1. Introduction

The concept of an Index Matrix (IM) was introduced in [2] and discusssed in a series of papers [2–7] and book [8]. In the extended form, it is defined as follows (see [8]).

Let $\mathcal{I}$ be again a fixed set of indices,

$$\mathcal{I}^n = \{\langle i_1, i_2, ..., i_n\rangle | (\forall j : 1 \le j \le n)(i_j \in \mathcal{I})\}$$

and

$$\mathcal{I}^* = \bigcup_{1\le n\le\infty} \mathcal{I}^n.$$

Let $\mathcal{X}$ be a fixed set of some objects. In the particular cases, they can be either real numbers, or only the numbers 0 or 1, or logical variables, propositions or predicates, etc.

Let operations $\circ, * : \mathcal{X} \times \mathcal{X} \to \mathcal{X}$ be fixed.

An Extended IM (EIM) with index sets K and L $(K, L \subset \mathcal{I}^*)$ and elements from set $\mathcal{X}$ is called the object (see, [8]):

$$[K, L, \{a_{k_i,l_j}\}] \equiv \begin{array}{c|ccccc} & l_1 & \dots & l_j & \dots & l_n \\ \hline k_1 & a_{k_1,l_1} & \cdots & a_{k_1,l_j} & \cdots & a_{k_1,l_n} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_i & a_{k_i,l_1} & \cdots & a_{k_i,l_j} & \cdots & a_{k_i,l_n} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_m & a_{k_m,l_1} & \cdots & a_{k_m,l_j} & \cdots & a_{k_m,l_n} \end{array},$$

where $K = \{k_1, k_2, ..., k_m\}$, $L = \{l_1, l_2, ..., l_n\}$, for $1 \leq i \leq m$, and $1 \leq j \leq n$: $a_{k_i,l_j} \in \mathcal{X}$.

For the EIMs (for brevity – only IMs) $A = [K, L, \{a_{k_i,l_j}\}], B = [P, Q, \{b_{p_r,q_s}\}]$, operations that are analogous to the usual matrix operations of addition and multiplication are defined, as well as other, specific ones. Some of them are the following:

Addition

$$A \oplus_{(\circ)} B = [K \cup P, L \cup Q, \{c_{t_u,v_w}\}],$$

where

$$c_{t_u,v_w} = \begin{cases} a_{k_i,l_j}, & \text{if } t_u = k_i \in K \text{ and } v_w = l_j \in L - Q \\ & \text{or } t_u = k_i \in K - P \text{ and } v_w = l_j \in L; \\ b_{p_r,q_s}, & \text{if } t_u = p_r \in P \text{ and } v_w = q_s \in Q - L \\ & \text{or } t_u = p_r \in P - K \text{ and } v_w = q_s \in Q; \\ a_{k_i,l_j} \circ b_{p_r,q_s}, & \text{if } t_u = k_i = p_r \in K \cap P \\ & \text{and } v_w = l_j = q_s \in L \cap Q \\ 0, & \text{otherwise} \end{cases}$$

Of course, here and below, if "$\circ$" is substituted by "+", then $a_{k_i,l_j} \circ b_{p_r,q_s} = a_{k_i,l_j} + b_{p_r,q_s}$, while, if "$\circ$" is "max" or min, then $a_{k_i,l_j} \circ b_{p_r,q_s} = \max(a_{k_i,l_j}, b_{p_r,q_s})$ or $a_{k_i,l_j} \circ b_{p_r,q_s} = \min(a_{k_i,l_j}, b_{p_r,q_s})$, respectively.

Termwise multiplication

$$A \otimes_{(\circ)} B = [K \cap P, L \cap Q, \{c_{t_u,v_w}\}],$$

where

$$c_{t_u,v_w} = a_{k_i,l_j} \circ b_{p_r,q_s},$$

for $t_u = k_i = p_r \in K \cap P$ and $v_w = l_j = q_s \in L \cap Q$.

Multiplication

$$A \odot_{(\circ,*)} B = [K \cup (P - L), Q \cup (L - P), \{c_{t_u,v_w}\}],$$

where

$$c_{t_u,v_w} = \begin{cases} a_{k_i,l_j}, & \text{if } t_u = k_i \in K \text{ and } v_w = l_j \in L - P - Q \\ b_{p_r,q_s}, & \text{if } t_u = p_r \in P - L - K \text{ and } v_w = q_s \in Q \\ \underset{l_j = p_r \in L \cap P}{\circ} a_{k_i,l_j} * b_{p_r,q_s}, & \text{if } t_u = k_i \in K \text{ and } v_w = q_s \in Q \\ 0, & \text{otherwise} \end{cases}$$

Obviously, when $\circ$ is substituted by $+$, the symbol $\underset{j}{\circ}$ is substituted by $\sum\limits_j$.

Structural subtraction

$$A \ominus B = [K - P, L - Q, \{c_{t_u,v_w}\}],$$

where "–" is the set–theoretic difference operation and

$$c_{t_u,v_w} = a_{k_i,l_j}, \text{ for } t_u = k_i \in K - P \text{ and } v_w = l_j \in L - Q.$$

The operation(s) in the sub-index of the operation between IMs, determine(s) the type of operation between the resultant IM-elements.

In the case of $(0,1)$-IM, $\circ, * \in \{\min, \max\}$.

2. Definition and properties of the new operation

Let $\mathcal{M}$ be the set of all IMs. Let

$$I_\emptyset = [\emptyset, \emptyset, \perp],$$

where symbol "$\perp$" denotes the lack of IM-elements.

Let, as above, we have the two IMs $A = [K, L, \{a_{k_i,l_j}\}]$ and $B = [P, Q, \{b_{p_r,q_s}\}]$. Then

$$A \circledwedge B = [K \cup P, L \cup Q, \{c_{t_u,v_w}\}],$$

where

$$c_{t_u,v_w} = \begin{cases} \langle a_{k_i,l_j}, \perp \rangle, & \text{if } t_u = k_i \in K \text{ and } v_w = l_j \in L - Q \\ & \text{or } t_u = k_i \in K - P \text{ and } v_w = l_j \in L; \\ \langle \perp, b_{p_r,q_s} \rangle, & \text{if } t_u = p_r \in P \text{ and } v_w = q_s \in Q - L \\ & \text{or } t_u = p_r \in P - K \text{ and } v_w = q_s \in Q; \\ \langle a_{k_i,l_j}, b_{p_r,q_s} \rangle, & \text{if } t_u = k_i = p_r \in K \cap P \\ & \text{and } v_w = l_j = q_s \in L \cap Q \\ \langle \perp, \perp \rangle, & \text{otherwise} \end{cases}$$

Obviously, this operation is not commutative, while, it is seen easily that the operation is associative and the validity of the equalities

$$A \circledwedge I_\emptyset = A = I_\emptyset \circledwedge A.$$

Therefore, it is valid

Theorem 1. $\langle \mathcal{M}, \circledwedge, I_\emptyset \rangle$ is a (non-commutative) monoid.

If we modify operation $\circ$ to the form:

$$\langle a, c \rangle \circ \langle b, c \rangle = \langle a \circ b, c \rangle,$$

then we can check that operation $\circledwedge$ satisfies equalities:

$$(A \oplus B) \circledwedge C = (A \circledwedge C) \oplus (B \circledwedge C),$$

$$(A \otimes B) \circledwedge C = (A \circledwedge C) \otimes (B \circledwedge C),$$

$$(A \odot B) \circledwedge C = (A \circledwedge C) \odot (B \circledwedge C),$$

$$(A \ominus B) \owedge C = (A \owedge C) \ominus (B \owedge C).$$

Let for every natural number $n \geq 2$ and for every n IMs $A_1, ..., A_n$:

$$\bigowedge_{i=1}^{2} A_i = A_1 \owedge A_2,$$

$$\bigowedge_{i=1}^{n} A_i = \left(\bigowedge_{i=1}^{n-1} A_i \right) \owedge A_n.$$

Let index set $\mathcal{I}$ and set $\mathcal{X}$ be fixed and let the EIMs $A_1, A_2, ..., A_n$ over both sets be given. In [8] the following operation is introduced.

Let for s $(1 \leq s \leq n)$:

$$(\forall p, q)(1 \leq p < q \leq n)(K^p \cap K^q = L^p \cap L^q = \emptyset) \qquad (*)$$

and

$$A_s = [K^s, L^s, \{a^s_{k_i,l_j}\}] = \begin{array}{c|ccccc} & l_{s,1} & \ldots & l_{s,j} & \ldots & l_{s,n_s} \\ \hline k_{s,1} & a_{k_{s,1},l_{s,1}} & \ldots & a_{k_{s,1},l_{s,j}} & \ldots & a_{k_{s,1},l_{s,n_s}} \\ \vdots & \vdots & \ldots & \vdots & \ldots & \vdots \\ k_{s,i} & a_{k_{s,i},l_{s,1}} & \ldots & a_{k_{s,i},l_{s,j}} & \ldots & a_{k_{s,i},l_{s,n_s}} \\ \vdots & \vdots & \ldots & \vdots & \ldots & \vdots \\ k_{s,m} & a_{k_{s,m},l_{s,1}} & \ldots & a_{k_{s,m},l_{s,j}} & \ldots & a_{k_{s,m},l_{s,n_s}} \end{array}.$$

Then, operation "composition" is defined by

$$\flat\{A_s | 1 \leq s \leq n\}$$

$$= [\bigcup_{s=1}^{n} K^s, \bigcup_{s=1}^{n} L^s, \{\langle c_{1,t_{1,u},v_{1,w}}, c_{2,t_{2,u},v_{2,w}}, ..., c_{n,t_{n,u},v_{n,w}} \rangle\}],$$

where for r $(1 \leq r \leq n)$:

$$c_{r,t_u,v_w} = \begin{cases} a_{r,k_i,l_j}, & \text{if } t_u = k_i \in K^r \text{ and } v_w = l_j \in L^r \\ \\ \bot, & \text{otherwise} \end{cases}$$

Let us omit condition (*). Then, e.g. by induction, we can see that it is valid

Theorem 2. For every IMs $A_1, A_2, ..., A_n$:

$$\flat\{A_s | 1 \leq s \leq n\} = \bigowedge_{i=1}^{n} A_i.$$

3. Conclusion

In future, the relations between the new oopertion and the other operatins, defined over IMs, will be studied. Also, in a next research, another operation, related to operation $\owedge$ will be discussed. It will have the opposite behaviour.

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