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REMARK ON THE INTUITIONISTIC FUZZY FORMS OF TWO CLASSICAL LOGIC AXIOMS. Part 1

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Abstract: It is checked which intuitionistic fuzzy implications satisfy the well-known logical tautologies

$$(p\&q) \to r = (p \to (q \to r)),$$

$$p \to q = (p \to (p \to q)).$$

Keywords: Intuitionistic fuzzy implication, Logical tautology.

In a series of research, we discuss the intuitionistic fuzzy forms of some classical logic axioms, and check their validity in the case of intuitionistic fuzziness.

Two of the well-known logical tautologies (see, e.g., [2]) are

$$(p\&q) \to r = (p \to (q \to r)), \tag{1}$$

$$p \to q = (p \to (p \to q)). \tag{2}$$

Here, we discuss their validity for the different cases of intuitionistic fuzzy implications. In [1], 138 of them are given.

Below, we determine which of these 138 intuitionistic fuzzy implications satisfy (1) and (2).

Theorem 1. Implications $\to_3, \to_4, \to_{11}, \to_{12}, \to_{14}, \to_{16}, \to_{18}, \to_{19}, \to_{20}, \to_{22}, \to_{23}, \to_{25}, \to_{26}, \to_{27}, \to_{28}, \to_{31}, \to_{32}, \to_{33}, \to_{41}, \to_{42}, \to_{43}, \to_{48}, \to_{56}, \to_{57}, \to_{74}, \to_{76}, \to_{77}, \to_{79}, \to_{81}, \to_{88}, \to_{97}$ satisfy (1).

Proof. Below, we prove that (1) is valid for implication $\to_3$. The rest assertions are proved by analogy. Let everywhere below, truth-values of p, q, r be:

$$V(p) = \langle a, b\rangle,$$

$$V(q) = \langle c, d\rangle,$$

$$V(r) = \langle e, f\rangle.$$

In [1], implication $\to_3$ is Gödel's implication, that has the form:

$$V(p \to_3 q) = \langle a, b\rangle \to_3 \langle c, d\rangle = \langle 1 - (1 - c).\text{sg}(a - c), d.\text{sg}(a - c)\rangle,$$

where

$$\text{sg}(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x \le 0 \end{cases}.$$

There it is defined that

$$V(p\&q) = \langle \min(a, c), \max(b, d)\rangle.$$

Therefore, the expression (1) has the form

$$(\langle a, b\rangle\&\langle c, d\rangle) \to_3 \langle e, f\rangle = \langle a, b\rangle \to_3 (\langle c, d\rangle \to_3 \langle e, f\rangle). \tag{3}$$

The left side has the form

$$(\langle a, b\rangle\&\langle c, d\rangle) \to_3 \langle e, f\rangle$$
$$= \langle \min(a, c), \max(b, d)\rangle \to_3 \langle e, f\rangle$$
$$= \langle 1 - (1 - e).\text{sg}(\min(a, c) - e), f.\text{sg}(\min(a, c) - e)\rangle.$$

The right side has the form

$$\langle a, b\rangle \to_3 (\langle c, d\rangle \to_3 \langle e, f\rangle)$$
$$= \langle a, b\rangle \to_3 \langle 1 - (1 - e).\text{sg}(c - e), f.\text{sg}(c - e)\rangle$$
$$= \langle 1 - (1 - (1 - (1 - e).\text{sg}(c - e))).\text{sg}(a - (1 - (1 - e).\text{sg}(c - e))),$$
$$f.\text{sg}(c - e).\text{sg}(a - (1 - (1 - e).\text{sg}(c - e)))\rangle$$
$$= \langle 1-(1-e).\text{sg}(c-e).\text{sg}(a-1+(1-e).\text{sg}(c-e)), f.\text{sg}(c-e).\text{sg}(a-1+(1-e).\text{sg}(c-e))\rangle.$$

Let

$$X \equiv 1 - (1 - e).\text{sg}(\min(a, c) - e) - 1 + (1 - e).\text{sg}(c - e).\text{sg}(a - 1 + (1 - e).\text{sg}(c - e)).$$

If $c \le e$, then $\min(a,c) \le c \le e$ and

$$X = 1 - (1-e).\text{sg}(\min(a,c) - e) - 1 + (1-e).\text{sg}(c-e).\text{sg}(a - 1 + (1-e).\text{sg}(c-e))$$
$$= 1 - 1 = 0.$$

If $c > e$, then

$$X = 1 - (1-e).\text{sg}(\min(a,c) - e) - 1 + (1-e).\text{sg}(a - 1 + (1-e))$$
$$= 1 - (1-e).\text{sg}(\min(a,c) - e) - 1 + (1-e).\text{sg}(a-e).$$

If $a \le e$, then $\min(a,c) \le a \le e$ and

$$X = 1 - 1 = 0.$$

If $a > e$, then $\min(a,c) > e$ and

$$X = 1 - (1-e) - 1 + (1-e) = 0.$$

Therefore, in all cases $X = 0$.

Let

$$Y \equiv f.\text{sg}(\min(a,c) - e) - f.\text{sg}(c-e).\text{sg}(a - 1 + (1-e).\text{sg}(c-e))$$

If $c \le e$, then $\min(a,c) \le c \le e$ and

$$Y = 0 - 0 = 0.$$

If $c > e$, then

$$Y = f.\text{sg}(\min(a,c) - e) - f.\text{sg}(a-e).$$

If $a \le e$, then $\min(a,c) \le a \le e$ and

$$Y = 0 - 0 = 0.$$

If $a > e$, then $\min(a,c) > e$ and

$$Y = f - f = 0.$$

Therefore, in all cases $Y = 0$. Hence, (3) is an equality for implication $\rightarrow_3$.

Theorem 2. Implications $\rightarrow_1, \rightarrow_2, \rightarrow_3, \rightarrow_4, \rightarrow_8, \rightarrow_{10}, \rightarrow_{11}, \rightarrow_{12}, \rightarrow_{14}, \rightarrow_{16}, \rightarrow_{17}, \rightarrow_{18}, \rightarrow_{19}, \rightarrow_{20}, \rightarrow_{22}, \rightarrow_{23}, \rightarrow_{25}, \rightarrow_{26}, \rightarrow_{27}, \rightarrow_{28}, \rightarrow_{30}, \rightarrow_{31}, \rightarrow_{32}, \rightarrow_{33}, \rightarrow_{36}, \rightarrow_{37}, \rightarrow_{39}, \rightarrow_{40}, \rightarrow_{41}, \rightarrow_{42}, \rightarrow_{43}, \rightarrow_{48}, \rightarrow_{51}, \rightarrow_{52}, \rightarrow_{54}, \rightarrow_{55}, \rightarrow_{56}, \rightarrow_{57}, \rightarrow_{59}, \rightarrow_{61}, \rightarrow_{67}, \rightarrow_{72}, \rightarrow_{73}, \rightarrow_{74}, \rightarrow_{76}, \rightarrow_{77}, \rightarrow_{78}, \rightarrow_{79}, \rightarrow_{80}, \rightarrow_{81}, \rightarrow_{86}, \rightarrow_{87}, \rightarrow_{88}, \rightarrow_{89}, \rightarrow_{91}, \rightarrow_{92}, \rightarrow_{95}, \rightarrow_{96}, \rightarrow_{97}, \rightarrow_{100}, \rightarrow_{105}, \rightarrow_{106}, \rightarrow_{109}, \rightarrow_{110}, \rightarrow_{114}, \rightarrow_{119}, \rightarrow_{120}$ satisfy (2).

References

[1] Atanassov, K., *On Intuitionistic Fuzzy Sets Theory*, Springer, Berlin, 2012.

[2] Kleene, S., *Mathematical Logic*, John Wiley & Sons, New York, 1967.