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INTUITIONISTIC FUZZY COGNITIVE MAPS AS A TOOL FOR INFLUENCE ANALYSIS OF THEIR CONSTITUENT FACTORS

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Abstract: Using the definition of the Intuitionistic Fuzzy Cognitive Map (IFCM), introduced in previous papers of the authors, they show that the IFCMs can be used as tools for analysis and risk evaluation.
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1. Introduction

The notion "cognitive map" has been introduced by the psychologist Tolman (1948) in his paper [1] as mental models (belief systems) of the way animals - including men - structure their environment. However, the cognitive approach to analysis of ill-structured situations was proposed by Axelrod [2] and Roberts [3] in 1976. Roberts was much more engaged into development of mathematical tools, while Axelrod drew attention to the methodology. A cognitive map is digraph $G(V, E)$ with vertex set V and arc set E. It is defined by the adjacency (or weight) matrix $W = [w_{i,j}]$, where $w_{i,j}$ is the weight of the arc $e_{i,j} \in E$. The vertices F_j of the cognitive map correspond to factors (concepts) that characterize a system (or a situation) and the arcs define causal connections between factors. In general, each factor represents a characteristic of the analyzed system, e.g. parameters, attributes, states, events, actions, values, goals, trends, components, resources. There are three possible types of causal relationships between each two factors F_i and F_j that describe the strength of influence going from one factor to the other. The weights of the arcs could be positive, $w_{i,j} > 0$, which means that an increase in the value of factor F_i leads to the increase of the value of factor F_j and a decrease of the value of factor F_i leads to the decrease of the value of factor F_j; the causality could be negative, $w_{i,j} < 0$, which means that an increase in the value of factor F_i leads to the decrease of the value of factor F_j, and vice versa.

In 1986 B. Kosko [4] enhanced the functions and power of cognitive maps by adding the following properties:

- arcs can take any real value in the interval $[-1, 1]$;
- nodes can take values in the set $\{-1, 1\}$ or in the set $\{0, 1\}$;
- nodes are time-valued;
- the value of each node at any moment is a function of the weighted sum of all its incoming nodes.

Kosko's definition is extended by S.M. Chen in [11] about ten years later.

The first idea for Intuitionistic Fuzzy Cognitive Map (IFCM) is given by S. K. A. De, R. Biswas and E. Roy in [12]. It is developed by E. I. Papageorgiou and D. K. Iakovidis in [13,14], and by the authors in [17]. Here, we give our definition of IFCM from [17], that is an extension of Papageorgiou and Iakovidis's one and show the way for using of the information from it for its factors analysis.

2. Definition of an intuitionistic fuzzy cognitive map

Below, we introduce the original definition of an IFCM from [17].

Let $\mathcal{C} = \{C_1, C_2, ..., C_n\}$ be a set of cognitive units and for every i ($i \in \{1, 2, ..., n\}$), $\mu_C(C_i)$ and $\nu_C(C_i)$ are degrees of validity and non-validity of the cognitive unit C_i.

Extending Chen's formal definitions of Fuzzy Cognitive Map (FCM, see [11]), we introduce the concept of an Intuitionistic FCM (IFCM) as the pair

$$IFCM = \langle C, E \rangle,$$

where

$$C = \{\langle C_i, \mu_C(C_i), \nu_C(C_i)\rangle | C_i \in \mathcal{C}\}$$

is an IFS and

$$E = [\mathcal{C}, \mathcal{C}, \{\langle \mu_E(e_{i,j}), \nu_E(e_{i,j})\rangle\}],$$

is an Intuitionistic Fuzzy Index Matrix (IFIM, see [8]) of incidence and for every $i, j \in \{1, 2, ..., n\}$, $\mu_E(e_{i,j})$ and $\nu_E(e_{i,j})$ are degrees of validity and non-validity of the oriented edge between neighbouring nodes $C_i, C_j \in \mathcal{C}$.

For every two cognitive units C_i and C_j that are connected with an edge $e_{i,j}$, we can introduce different criteria for correctness, e.g. if C_i is higher than C_j (i.e., $\langle \mu_C(C_i), \nu_C(C_i)\rangle \geq \langle \mu_C(C_j), \nu_C(C_j)\rangle$), then

1 (top-down-max-min) $\langle \mu_C(C_i), \nu_C(C_i)\rangle \vee \langle \mu_E(e_{i,j}), \nu_E(e_{i,j})\rangle \geq \langle \mu_C(C_j), \nu_C(C_j)\rangle$;

2 (top-down-average) $\langle \mu_C(C_i), \nu_C(C_i) \rangle @ \langle \mu_E(e_{i,j}), \nu_E(e_{i,j}) \rangle \geq \langle \mu_C(C_j), \nu_C(C_j) \rangle$;

3 (top-down-min-max) $\langle \mu_C(C_i), \nu_C(C_i) \rangle \wedge \langle \mu_E(e_{i,j}), \nu_E(e_{i,j}) \rangle \geq \langle \mu_C(C_j), \nu_C(C_j) \rangle$;

4 (down-top-max-min) $\langle \mu_C(C_i), \nu_C(C_i) \rangle \wedge \langle \mu_E(e_{i,j}), \nu_E(e_{i,j}) \rangle \leq \langle \mu_C(C_j), \nu_C(C_j) \rangle$;

5 (down-top-average) $\langle \mu_C(C_i), \nu_C(C_i) \rangle @ \langle \mu_E(e_{i,j}), \nu_E(e_{i,j}) \rangle \leq \langle \mu_C(C_j), \nu_C(C_j) \rangle$;

6 (down-top-min-max) $\langle \mu_C(C_i), \nu_C(C_i) \rangle \vee \langle \mu_E(e_{i,j}), \nu_E(e_{i,j}) \rangle \leq \langle \mu_C(C_j), \nu_C(C_j) \rangle$.

Other criteria also are possible.

3. Intuitionistic fuzzy cognitive maps as tools for analysis of its factors

The definition of an IFCM in the above form is more general than it is necessary for the present research. In it, the term “cognitive unit”’ is used, while in the present case we need its partial case of “factor” in Axelrod-Kosko-Chen’s sense. By this reason, below, we change the used in the definition symbol C with F. Now, we can use ideas from our research [18], where some properties of the factors are discussed. These properties are influenced by Silov’s fuzzy cognitive maps, discussed in [5].

Below, we use the concept of an index matrix [6, 7, 10], that is an extension of the ordinary matrix. Let I be a fixed set of indices and $\mathcal{R}$ be the set of the real numbers. By IM with index sets K and L $(K, L \subset I)$, we denoted the object:

$$[K, L, \{a_{k_i,l_j}\}] \equiv \begin{array}{c|cccc} & l_1 & l_2 & \dots & l_n \\ \hline k_1 & a_{k_1,l_1} & a_{k_2,l_2} & \dots & a_{k_1,l_n} \\ k_2 & a_{k_2,l_1} & a_{k_2,l_2} & \dots & a_{k_2,l_n} \\ \vdots & & & & \\ k_m & a_{k_m,l_1} & a_{k_m,l_2} & \dots & a_{k_m,l_n} \end{array},$$

where $K = \{k_1, k_2, ..., k_m\}$, $L = \{l_1, l_2, ..., l_n\}$, for $1 \leq i \leq m$, and $1 \leq j \leq n : a_{k_i,l_j} \in \mathcal{R}$.

In [6–8, 15, 16], different operations, relations and operators are defined over IMs. For the needs of the present research, we will introduce the definitions of some of them.

Let us have set

$$\mathcal{F} = \{F_1, ..., F_n\}$$

of factors. We can construct the IM of influence of factor F_i over factor F_j with the form

$$\begin{array}{c|ccc} & F_1 & \dots & F_n \\ \hline F_1 & \langle z_{1,1}, \overline{z}_{1,1} \rangle & \dots & \langle z_{1,n}, \overline{z}_{1,n} \rangle \\ \vdots & \vdots & \dots & \vdots \\ F_n & \langle z_{n,1}, \overline{z}_{n,1} \rangle & \dots & \langle z_{n,n}, \overline{z}_{n,n} \rangle \end{array},$$

where $z_{i,j}, \overline{z}_{i,j} \in [-1, 1]$ determine the degrees of positive and negative influences of factor F_i over factor F_j.

Let

$$\mathcal{F}^i = \mathcal{F} - \{F_i\}.$$

First, we must metion that all notations related to the intuitionistic fuzziness are used from [9, 10]. For the text below, we must mention that the ordered pair $\langle a, b\rangle$ is an Intuitionistic Fuzzy Pair (IFP) if and only if (iff) $a, b, a + b \in [0, 1]$. For two IFPs $\langle a, b\rangle$ and $\langle c, d\rangle$:

$$\langle a, b\rangle \leq \langle c, d\rangle \text{ iff } a \leq c \text{ and } b \geq d,$$

$$\neg\langle a, b\rangle = \langle b, a\rangle,$$

$$\langle a, b\rangle \wedge \langle c, d\rangle = \langle \min(a, c), \max(b, d)\rangle,$$

$$\langle a, b\rangle \vee \langle c, d\rangle = \langle \max(a, c), \min(b, d)\rangle.$$

Second, let us have the pair $\langle z_{i,j}, \overline{z}_{i,j}\rangle$. We can transform it to pair $\langle y_{i,j}, \overline{y}_{i,j}\rangle$ using formula

$$\langle y_{i,j}, \overline{y}_{i,j}\rangle = \begin{cases} \langle \frac{z_{i,j}+1}{2}, \frac{\overline{z}_{i,j}+1}{2}\rangle, & \text{if } z_{i,j} < 0 \text{ and } \overline{z}_{i,j} < 0 \\ \langle \frac{z_{i,j}+1}{2}, \frac{\overline{z}_{i,j}}{2}\rangle, & \text{if } z_{i,j} < 0 \text{ and } \overline{z}_{i,j} \geq 0 \\ \langle \frac{z_{i,j}}{2}, \frac{\overline{z}_{i,j}+1}{2}\rangle, & \text{if } z_{i,j} \geq 0 \text{ and } \overline{z}_{i,j} < 0 \\ \langle \frac{z_{i,j}}{2}, \frac{\overline{z}_{i,j}}{2}\rangle, & \text{if } z_{i,j} \geq 0 \text{ and } \overline{z}_{i,j} \geq 0 \end{cases}.$$

For every $z_{i,j}, \overline{z}_{i,j} \in [-1, 1]$, the pair $\langle y_{i,j}, \overline{y}_{i,j}\rangle$ is an IFP.

Really, if $z_{i,j} < 0$ and $\overline{z}_{i,j} < 0$, then

$$y_{i,j} + \overline{y}_{i,j} = \frac{z_{i,j} + 1}{2} + \frac{\overline{z}_{i,j} + 1}{2} = \frac{z_{i,j} + 1 + \overline{z}_{i,j} + 1}{2} \geq 0$$

and

$$y_{i,j} + \overline{y}_{i,j} = \frac{z_{i,j} + 1 + \overline{z}_{i,j} + 1}{2} \leq 1.$$

Therefore, $\langle y_{i,j}, \overline{y}_{i,j}\rangle$ is an IFP.

By similar way are proved the other three possibilities.

In this paper, an intuitionistic fuzzy interpretation of the above mentioned systemic indicators is proposed. A full correspondence between the formulas in Silov's fuzzy cognitive maps and intuitionistic fuzzy formulas given below.

The Intuitionistic Fuzzy Coefficient of Influence (IFCI) of factor F_i over factor F_j has the form

$$IFCI(F_i, F_j) = \langle \text{sg}(y_{i,j} - \overline{y}_{i,j}).(y_{i,j} - \overline{y}_{i,j}), \text{sg}(\overline{y}_{i,j} - y_{i,j}).(\overline{y}_{i,j} - y_{i,j})\rangle,$$

where function sg is defined by

$$sg(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \le 0 \end{cases}$$

The Intuitionistic Fuzzy Coefficient of Joint Influence (IFCJI) between factors F_i and F_j has one of the following forms

$$\begin{array}{l} IFCJI^{opt}(F_i, F_j) = \langle \max(y_{i,j}, y_{j,i}), \min(\overline{y}_{i,j}, \overline{y}_{j,i})\rangle \\ IFCJI^{opt-av}(F_i, F_j) = \langle \max(\frac{y_{i,j}+1-\overline{y}_{j,i}}{2}, \frac{y_{j,i}+1-\overline{y}_{i,j}}{2}), \frac{\overline{y}_{i,j}+\overline{y}_{j,i}}{2}\rangle \\ IFCJI^{av}(F_i, F_j) = \langle \frac{y_{i,j}+y_{j,i}}{2}, \frac{\overline{y}_{j,i}+\overline{y}_{i,j}}{2}\rangle \\ IFCJI^{pes-av}(F_i, F_j) = \langle \frac{y_{i,j}+y_{j,i}}{2}, \max(\frac{\overline{y}_{i,j}+1-y_{j,i}}{2}, \frac{\overline{y}_{j,i}+1-y_{i,j}}{2})\rangle \\ IFCJI^{pes}(F_i, F_j) = \langle \min(y_{i,j}, y_{j,i}), \max(\overline{y}_{i,j}, \overline{y}_{j,i})\rangle \end{array}.$$

The Consonance of the Influence (CI) of factor F_i over factor F_j has the form

$$CI(F_i, F_j) = \langle \frac{\max(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\min(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}\rangle.$$

The Consonance of the Joint Influence (CJI) of factors F_i and F_j has one of the three forms:

$$CJI^{opt}(F_i, F_j) = \langle \max(\frac{\max(y_{i,j}, \overline{y}_{i,j}) + \max(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}},$$

$$\frac{\max(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\max(y_{j,i}, \overline{y}_{j,i})}{y_{j,i} + \overline{y}_{j,i}}),$$

$$\min(\frac{\min(y_{i,j}, \overline{y}_{i,j}) + \min(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}}, \frac{\min(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\min(y_{j,i}, \overline{y}_{j,i})}{y_{j,i} + \overline{y}_{j,i}})\rangle,$$

$$CJI^{av-opt}(F_i, F_j) = \langle \frac{\max(y_{i,j}, \overline{y}_{i,j}) + \max(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}},$$

$$\frac{\min(y_{i,j}, \overline{y}_{i,j}) + \min(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}}\rangle,$$

$$CJI^{av-pes}(F_i, F_j) = \langle \frac{\min(y_{i,j}, \overline{y}_{i,j}) + \min(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}},$$

$$\frac{\max(y_{i,j}, \overline{y}_{i,j}) + \max(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}}\rangle,$$

$$CJI^{pes}(F_i, F_j) = \langle \min(\frac{\min(y_{i,j}, \overline{y}_{i,j}) + \min(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}},$$
$$\frac{\min(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\min(y_{j,i}, \overline{y}_{j,i})}{y_{j,i} + \overline{y}_{j,i}}),$$
$$\max(\frac{\max(y_{i,j}, \overline{y}_{i,j}) + \max(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}}, \frac{\max(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\max(y_{j,i}, \overline{y}_{j,i})}{y_{j,i} + \overline{y}_{j,i}})\rangle.$$

The Dissonance of the Influence (DI) of factor F_i over factor F_j has the form

$$DI(F_i, F_j) = \langle \frac{\min(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\max(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}} \rangle.$$

The Dissonance of the Joint Influence (DJI) of factors F_i and F_j has one of the three forms:

$$DJI^{opt}(F_i, F_j) = \langle \min(\frac{\min(y_{i,j}, \overline{y}_{i,j}) + \min(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}},$$
$$\frac{\min(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\min(y_{j,i}, \overline{y}_{j,i})}{y_{j,i} + \overline{y}_{j,i}}),$$
$$\max(\frac{\max(y_{i,j}, \overline{y}_{i,j}) + \max(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}}, \frac{\max(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\max(y_{j,i}, \overline{y}_{j,i})}{y_{j,i} + \overline{y}_{j,i}})\rangle,$$
$$DJI^{av-opt}(F_i, F_j) = \langle \frac{\min(y_{i,j}, \overline{y}_{i,j}) + \min(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}},$$
$$\frac{\max(y_{i,j}, \overline{y}_{i,j}) + \max(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}}\rangle,$$
$$DJI^{av-pes}(F_i, F_j) = \langle \frac{\max(y_{i,j}, \overline{y}_{i,j}) + \max(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}},$$
$$\frac{\min(y_{i,j}, \overline{y}_{i,j}) + \min(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}}\rangle,$$
$$DJI^{pes}(F_i, F_j) = \langle \max(\frac{\max(y_{i,j}, \overline{y}_{i,j}) + \max(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}},$$
$$\frac{\max(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\max(y_{j,i}, \overline{y}_{j,i})}{y_{j,i} + \overline{y}_{j,i}}),$$
$$\min(\frac{\min(y_{i,j}, \overline{y}_{i,j}) + \min(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}}, \frac{\min(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\min(y_{j,i}, \overline{y}_{j,i})}{y_{j,i} + \overline{y}_{j,i}})\rangle,$$

The Consonance of the Joint Influence of factor F_i over all Other Factors (CJIOF), i.e., the factors from $\mathcal{F}^i$, has one of the three forms:

$$CJIOF^{opt}(F_i, \mathcal{F}^i) = \langle \max_{1 \le j \le n, j \ne i} y_{i,j}, \min_{1 \le j \le n, j \ne i} \overline{y}_{i,j} \rangle,$$

$$CJIOF^{av}(F_i, \mathcal{F}^i) = \langle \frac{1}{n-1} \sum_{1 \le j \le n, j \ne i} y_{i,j}, \frac{1}{n-1} \sum_{1 \le j \le n, j \ne i} \overline{y}_{i,j} \rangle,$$

$$CJIOF^{pes}(F_i, \mathcal{F}^i) = \langle \min_{1 \le j \le n, j \ne i} y_{i,j}, \max_{1 \le j \le n, j \ne i} \overline{y}_{i,j} \rangle.$$

The Dissonance of the Joint Influence of factor F_i over all Other Factors (DJIOF), i.e., the factors from $\mathcal{F}^i$, has one of the three forms:

$$DJIOF^{opt}(F_i, \mathcal{F}^i) = \langle \min_{1 \le j \le n, j \ne i} \overline{y}_{i,j}, \max_{1 \le j \le n, j \ne i} y_{i,j} \rangle.$$

$$DJIOF^{av}(F_i, \mathcal{F}^i) = \langle \frac{1}{n-1} \sum_{1 \le j \le n, j \ne i} \overline{y}_{i,j}, \frac{1}{n-1} \sum_{1 \le j \le n, j \ne i} y_{i,j} \rangle,$$

$$DJIOF^{pes}(F_i, \mathcal{F}^i) = \langle \max_{1 \le j \le n, j \ne i} \overline{y}_{i,j}, \min_{1 \le j \le n, j \ne i} y_{i,j} \rangle.$$

The Consonance of the Joint Influence of all Factors Excluding factor F_i (CJIFE), has one of the three forms:

$$CJIOF^{opt}(\mathcal{F}^i, F_i) = \langle \max_{1 \le j \le n, j \ne i} y_{j,i}, \min_{1 \le j \le n, j \ne i} (\overline{y}_{j,i}) \rangle,$$

$$CJIOF^{av}(\mathcal{F}^i, F_i) = \langle \frac{1}{n-1} \sum_{1 \le j \le n, j \ne i} y_{j,i}, \frac{1}{n-1} \sum_{1 \le j \le n, j \ne i} \overline{y}_{j,i} \rangle,$$

$$CJIOF^{pes}(\mathcal{F}^i, F_i) = \langle \min_{1 \le j \le n, j \ne i} y_{j,i}, \max_{1 \le j \le n, j \ne i} \overline{y}_{j,i} \rangle.$$

The Dissonance of the Joint Influence of factor F_i over all Other Factors (DJIOF), i.e., the factors from $\mathcal{F}^i$, has one of the three forms:

$$DJIOF^{opt}(\mathcal{F}^i, F_i) = \langle \min_{1 \le j \le n, j \ne i} \overline{y}_{j,i}, \max_{1 \le j \le n, j \ne i} (y_{j,i}) \rangle.$$

$$DJIOF^{av}(\mathcal{F}^i, F_i) = \langle \frac{1}{n-1} \sum_{1 \le j \le n, j \ne i} \overline{y}_{j,i}, \frac{1}{n-1} \sum_{1 \le j \le n, j \ne i} y_{j,i} \rangle,$$

$$DJIOF^{pes}(\mathcal{F}^i, F_i) = \langle \max_{1 \le j \le n, j \ne i} \overline{y}_{j,i}, \min_{1 \le j \le n, j \ne i} y_{j,i} \rangle.$$

The (Total) Influence of factor F_i over all Other Factors (IFOFs), has one of the two forms:

$$IFOFs^{opt}(F_i, \mathcal{F}^i) = \langle \max_{1 \le j \le n, j \ne i} (y_{i,j}, \overline{y}_{i,j}), \min_{1 \le j \le n, j \ne i} (y_{i,j}, \overline{y}_{i,j}) \rangle,$$

$$IFOFs^{pes}(F_i, \mathcal{F}^i) = \langle \min_{1\leq j\leq n, j\neq i} (y_{i,j}, \overline{y}_{i,j}), \max_{1\leq j\leq n, j\neq i} (y_{i,j}, \overline{y}_{i,j})\rangle,$$

The (Total) Influence of the factors, different than factor F_i over factor F_i all Other Factors (IFsOF), has one of the two forms:

$$IFsOF^{opt}(\mathcal{F}^i, F_i) = \langle \max_{1\leq j\leq n, j\neq i} (y_{j,i}, \overline{y}_{j,i}), \min_{1\leq j\leq n, j\neq i} (y_{j,i}, \overline{y}_{j,i})\rangle,$$

$$IFsOF^{pes}(\mathcal{F}^i, F_i) = \langle \min_{1\leq j\leq n, j\neq i} (y_{j,i}, \overline{y}_{j,i}), \max_{1\leq j\leq n, j\neq i} (y_{j,i}, \overline{y}_{j,i})\rangle,$$

The Consonance of the Joint Influence of factor F_i and all Other Factors ($CJI\&OF$), i.e., the factors from $\mathcal{F}^i$, has one of the three forms:

$$CJI\&OF^{opt}(F_i, \mathcal{F}^i) = \langle \max_{1\leq j\leq n, j\neq i} (y_{i,j}, y_{j,i}), \min_{1\leq j\leq n, j\neq i} (\overline{y}_{i,j}, \overline{y}_{j,i})\rangle,$$

$$CJI\&OF^{av}(F_i, \mathcal{F}^i) = \langle \frac{1}{2(n-1)} \sum_{1\leq j\leq n, j\neq i} (y_{i,j} + y_{j,i}), \frac{1}{2(n-1)} \sum_{1\leq j\leq n, j\neq i} (\overline{y}_{i,j} + \overline{y}_{j,i})\rangle,$$

$$CJI\&OF^{pes}(F_i, \mathcal{F}^i) = \langle \min_{1\leq j\leq n, j\neq i} (y_{i,j}, y_{j,i}), \max_{1\leq j\leq n, j\neq i} (\overline{y}_{i,j}, \overline{y}_{j,i})\rangle.$$

The Dissonance of the Joint Influence of factor F_i and all Other Factors ($DJI\&OF$), i.e., the factors from $\mathcal{F}^i$, has one of the three forms:

$$DJI\&OF^{opt}(F_i, \mathcal{F}^i) = \langle \min_{1\leq j\leq n, j\neq i} (\overline{y}_{i,j}, \overline{y}_{j,i}), \max_{1\leq j\leq n, j\neq i} (y_{i,j}, y_{j,i})\rangle.$$

$$DJI\&OF^{av}(F_i, \mathcal{F}^i) = \langle \frac{1}{2(n-1)} \sum_{1\leq j\leq n, j\neq i} (\overline{y}_{i,j} + \overline{y}_{j,i}), \frac{1}{2(n-1)} \sum_{1\leq j\leq n, j\neq i} (y_{i,j} + y_{j,i})\rangle,$$

$$DJI\&OF^{pes}(F_i, \mathcal{F}^i) = \langle \max_{1\leq j\leq n, j\neq i} (\overline{y}_{i,j}, \overline{y}_{j,i}), \min_{1\leq j\leq n, j\neq i} (y_{i,j}, y_{j,i})\rangle.$$

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