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INTUITIONISTIC FUZZY INTERPRETATION OF THE JOINT INFLUENCE BETWEEN THE KEY FACTORS IN FUZZY COGNITIVE MAPS

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Abstract: Fuzzy cognitive maps are powerful tool for modeling of complex socio-technical systems. The informed decision making process in the uncertain environments of such systems requires a thorough understanding of the key characteristics that define their behavior. In this respect, the proposed intuitionistic fuzzy interpretation of the entire set of joint influences between the factors in fuzzy cognitive maps allows an enhanced description and analysis of the driving forces and system's dynamic.
Keywords: Fuzzy cognitive map, Intuituonistic fuzzy pair.

1. Introduction

Almost all systems found in nature and society are complex systems. Socio-technical systems, a widespread class of complex systems deserve particular attention due to their special characteristics. As stated in [11], complex systems can have all or some of the following characteristics, several of which may overlap: adaptability, emergence, self-organization, attractors, self-organized criticality, chaos, non-linearity, phase transition, power laws. In addition, the human factor in socio-technical systems is a matter of serious concern. A socio-technical system comprises of interacting and interdependent social/institutional and physical/technological parts, characterized by inputs, processes/actions, and outputs/products [9]. It can also be described as interacting and coupled networks of components, which are connected through additional information networks. One layer of the system includes the physical/technological components and the other layer includes the social/institutional ones. The relationships between the constituent components and subsystems of these three networks are not always clearly understood, which is the major challenge in analyzing future global risks and shocks. Thus, the understanding of key characteristics of complex systems is crucial for anticipating events that may require appropriate corrective measures and identifying where those interventions should be applied with maximum efficiency.

2. Cognitive approach to socio-technical systems

In real-life systems, the decision making process is an indispensable part of the integrated management activities. Moreover, the decision theory has become an independent area of scientific research during the last century. In order to cope with the emerging complex realities, decision theory uses different methods of mathematics, psychology, sociology and informatics. Any scientific method of studying complex systems is based on some kind of modeling and computer simulation. One of the powerful new approaches of modern decision theory is the cognitive simulation. The cognitive approach to modeling and simulation is aimed at development of mathematical models and methods suitable to support the entire decision making process, taking into account the cognitive abilities of decision makers and stakeholders (e.g. the human factor).

As a rule, when building a models or analyzing the behavior of socio-technical systems, scientists and engineers always face weakly-formalized and/ or ill-structured problems. The concept of ill-structured problem (situation) was introduced by H. Simon [13]. It means that the building elements of the system as well as their relationships have both quantitative and qualitative nature. On the other hand the basic parameters of such situation (values of the factors, strength of causal relationships between the factors) are very often not quantitative but have qualitative characteristics - fuzzy values, intervals, linguistic variables and estimates.

The modern methods to the modeling, analysis and control of ill-structured and weakly formalized problems are based on the broad implementation of cognitive maps. The notion "cognitive map" has been introduced by the psychologist Tolman (1948) in his paper [14] as mental models (belief systems) of the way animals - including men - structure their environment. However, the cognitive approach to analysis of ill-structured situations was proposed by Axelrod [6] and Roberts [12] in 1976. Roberts was much more engaged into development of mathematical tools, while Axelrod drew attention to the methodology. A cognitive map is digraph $G(V, E)$ with vertex set V and arc set E. It is defined by the adjacency (or weight) matrix $W = [w_{i,j}]$, where $w_{i,j}$ is the weight of the arc $e_{i,j} \in E$. The vertices F_j of the cognitive map correspond to factors (concepts) that characterize a system (or a situation) and the arcs define causal connections between factors. In general, each factor represents a characteristic of the analyzed system, e.g. parameters, attributes, states, events, actions, values, goals, trends, components, resources. There are three possible types of causal relationships between each two factors F_i and F_j that describe the strength of influence going from one factor to the other. The weights of the arcs could be positive, $w_{i,j} > 0$, which means that an increase in the value of factor F_i leads to the increase of the value of factor F_j and a decrease of the value of factor F_i leads to the decrease of the value of factor F_j; the causality could be negative, $w_{i,j} < 0$, which means that an increase in the value of factor F_i leads to the decrease of the value of factor F_j, and vice versa.

3. Intuitionistic fuzzy estimations of the influences among factors

In 1986, B. Kosko [10] enhanced the functions and power of cognitive maps by adding the following properties:

- arcs can take any real value in the interval $[-1, 1]$;
- nodes can take values in the set $\{-1, 1\}$ or in the set $\{0, 1\}$;
- nodes are time-valued;
- the value of each node at any moment is a function of the weighted sum of all its incoming nodes.

Thus, fuzziness enters the cognitive maps through the values of the arc strengths and gives the opportunity to capture subtleties in the causal relationships that might exist. In the past decades after this pioneering work, fuzzy cognitive maps have been used by scientist and engineers to solve different ill-structured problems in many fields [8].

A unique approach to analysis of complex systems with fuzzy cognitive maps has been developed by Silov [7]. In addition to the weights and signs of the arcs, formulas for consonances and dissonances inherent to the causal relationships between all factors F_i have been defined. On the basis of the consonances and dissonances the matrices with the systemic indicators should be formed and used to more detailed analysis of the influences in the fuzzy cognitive map. The necessary prerequisite to perform this analysis is to calculate the cognitive matrix of transitive closure Z, which reflects all direct and indirect causal influences between the factors. The elements of this matrix are pairs $\langle z_{i,j}, \overline{z}_{i,j}\rangle$.

In this paper, we use the concept of an index matrix [1–5], that is an extension of the ordinary matrix.

Let us have set

$$\mathcal{F} = \{F_1, ..., F_n\}$$

of factors. We can construct the index matrix of influence of factor F_i over factor F_j with the form

$$\begin{array}{c|ccc} & F_1 & \dots & F_n \\ \hline F_1 & \langle z_{1,1}, \overline{z}_{1,1}\rangle & \dots & \langle z_{1,n}, \overline{z}_{1,n}\rangle \\ \vdots & \vdots & \dots & \vdots \\ F_n & \langle z_{n,1}, \overline{z}_{n,1}\rangle & \dots & \langle z_{n,n}, \overline{z}_{n,n}\rangle \end{array},$$

where $z_{i,j}, \overline{z}_{i,j} \in [-1, 1]$.

Let

$$\mathcal{F}^i = \mathcal{F} - \{F_i\}.$$

First, we must mention that all notations related to the intuitionistic fuzziness are used from [2, 5]. For the text below, we must mention that the ordered pair $\langle a, b\rangle$ is an Intuitionistic Fuzzy Pair (IFP) if and only if (iff) $a, b, a + b \in [0, 1]$. For two IFPs $\langle a, b\rangle$ and $\langle c, d\rangle$:

$$\langle a, b\rangle \leq \langle c, d\rangle \text{ iff } a \leq c \text{ and } b \geq d,$$

$$\neg\langle a,b\rangle = \langle b,a\rangle,$$
$$\langle a,b\rangle \wedge \langle c,d\rangle = \langle \min(a,c), \max(b,d)\rangle,$$
$$\langle a,b\rangle \vee \langle c,d\rangle = \langle \max(a,c), \min(b,d)\rangle.$$

Second, let us have the pair $\langle z_{i,j}, \overline{z}_{i,j}\rangle$. We can transform it to pair $\langle y_{i,j}, \overline{y}_{i,j}\rangle$ using formula

$$\langle y_{i,j}, \overline{y}_{i,j}\rangle = \begin{cases} \langle \frac{z_{i,j}+1}{2}, \frac{\overline{z}_{i,j}+1}{2}\rangle, & \text{if } z_{i,j} < 0 \text{ and } \overline{z}_{i,j} < 0 \\ \langle \frac{z_{i,j}+1}{2}, \frac{\overline{z}_{i,j}}{2}\rangle, & \text{if } z_{i,j} < 0 \text{ and } \overline{z}_{i,j} \geq 0 \\ \langle \frac{z_{i,j}}{2}, \frac{\overline{z}_{i,j}+1}{2}\rangle, & \text{if } z_{i,j} \geq 0 \text{ and } \overline{z}_{i,j} < 0 \\ \langle \frac{z_{i,j}}{2}, \frac{\overline{z}_{i,j}}{2}\rangle, & \text{if } z_{i,j} \geq 0 \text{ and } \overline{z}_{i,j} \geq 0 \end{cases}.$$

Proposition 1. For every $z_{i,j}, \overline{z}_{i,j} \in [-1, 1]$, the pair $\langle y_{i,j}, \overline{y}_{i,j}\rangle$ is an IFP.
Proof. Let

$$X = y_{i,j} + \overline{y}_{i,j}.$$

If $z_{i,j} < 0$ and $\overline{z}_{i,j} < 0$, then

$$X = \frac{z_{i,j}+1}{2} + \frac{\overline{z}_{i,j}+1}{2} = \frac{z_{i,j}+1+\overline{z}_{i,j}+1}{2} \geq 0$$

and

$$X = \frac{z_{i,j}+1+\overline{z}_{i,j}+1}{2} \leq 1.$$

Therefore, $\langle y_{i,j}, \overline{y}_{i,j}\rangle$ is an IFP.

By similar way, all other assertions are proved.

In this paper, an intuitionistic fuzzy interpretation of the above mentioned systemic indicators is proposed. A full correspondence between the formulas in Silov's fuzzy cognitive maps and intuitionistic fuzzy formulas is given below.

The Intuitionistic Fuzzy Coefficient of Joint Influence (IFCJI) between factors F_i and F_j has one of the following forms

$$\begin{array}{l} IFCJI^{opt}(F_i,F_j) = \langle \max(y_{i,j}, y_{j,i}), \min(\overline{y}_{i,j}, \overline{y}_{j,i})\rangle \\ IFCJI^{opt-av}(F_i,F_j) = \langle \max(\frac{y_{i,j}+1-\overline{y}_{j,i}}{2}, \frac{y_{j,i}+1-\overline{y}_{i,j}}{2}), \frac{\overline{y}_{i,j}+\overline{y}_{j,i}}{2}\rangle \\ IFCJI^{av}(F_i,F_j) = \langle \frac{y_{i,j}+y_{j,i}}{2}, \frac{\overline{y}_{j,i}+\overline{y}_{i,j}}{2}\rangle \\ IFCJI^{pes-av}(F_i,F_j) = \langle \frac{y_{i,j}+y_{j,i}}{2}, \max(\frac{\overline{y}_{i,j}+1-y_{j,i}}{2}, \frac{\overline{y}_{j,i}+1-y_{i,j}}{2})\rangle \\ IFCJI^{pes}(F_i,F_j) = \langle \min(y_{i,j}, y_{j,i}), \max(\overline{y}_{i,j}, \overline{y}_{j,i})\rangle \end{array}.$$

Proposition 2. For every two factors F_i and F_j,
(a) their IFCJIs are IFPs,
(b) it is valid that

$$IFCJI^{pes}(F_i,F_j) \vee IFCJI^{pes-av}(F_i,F_j)$$

$$\leq IFCJI^{av}(F_i, F_j)$$
$$\leq IFCJI^{opt-av}(F_i, F_j) \wedge IFCJI^{opt}(F_i, F_j).$$

The Consonance of the Influence (CI) of factor F_i over factor F_j has the form

$$CI(F_i, F_j) = \langle \frac{\max(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\min(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}} \rangle.$$

Proposition 3. For every two factors F_i and F_j, their CI $CI(F_i, F_j)$ is an IFP.

The Consonance of the Joint Influence (CJI) of factors F_i and F_j has one of the four forms:

$$CJI^{opt}(F_i, F_j) = \langle \max(\frac{\max(y_{i,j}, \overline{y}_{i,j}) + \max(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}},$$
$$\frac{\max(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\max(y_{j,i}, \overline{y}_{j,i})}{y_{j,i} + \overline{y}_{j,i}}),$$
$$\min(\frac{\min(y_{i,j}, \overline{y}_{i,j}) + \min(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}}, \frac{\min(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\min(y_{j,i}, \overline{y}_{j,i})}{y_{j,i} + \overline{y}_{j,i}})\rangle,$$
$$CJI^{av-opt}(F_i, F_j) = \langle \frac{\max(y_{i,j}, \overline{y}_{i,j}) + \max(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}},$$
$$\frac{\min(y_{i,j}, \overline{y}_{i,j}) + \min(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}} \rangle,$$
$$CJI^{av-pes}(F_i, F_j) = \langle \frac{\min(y_{i,j}, \overline{y}_{i,j}) + \min(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}},$$
$$\frac{\max(y_{i,j}, \overline{y}_{i,j}) + \max(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}} \rangle,$$
$$CJI^{pes}(F_i, F_j) = \langle \min(\frac{\min(y_{i,j}, \overline{y}_{i,j}) + \min(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}},$$
$$\frac{\min(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\min(y_{j,i}, \overline{y}_{j,i})}{y_{j,i} + \overline{y}_{j,i}}),$$
$$\max(\frac{\max(y_{i,j}, \overline{y}_{i,j}) + \max(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}}, \frac{\max(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\max(y_{j,i}, \overline{y}_{j,i})}{y_{j,i} + \overline{y}_{j,i}})\rangle.$$

Proposition 4. For every two factors F_i and F_j,
(a) their CJIs are IFPs,
(b) it is valid that

$$CJI^{pes}(F_i, F_j) \leq CJI^{av-pes}(F_i, F_j) \leq CJI^{av-opt}(F_i, F_j) \leq CJI^{opt}(F_i, F_j).$$

Obviously, for every i, j:

$$CJI^{opt}(F_i, F_j) = \neg CJI^{pes}(F_i, F_j),$$

$$CJI^{av-opt}(F_i, F_j) = \neg CJI^{av-pes}(F_i, F_j).$$

The Dissonance of the Influence (DI) of factor F_i over factor F_j has the form

$$DI(F_i, F_j) = \langle \frac{\min(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\max(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}} \rangle.$$

Proposition 5. For every two factors F_i and F_j, their DI $DI(F_i, F_j)$ is an IFP.

The Dissonance of the Joint Influence (DJI) of factors F_i and F_j has one of the four forms:

$$DJI^{opt}(F_i, F_j) = \langle \min(\frac{\min(y_{i,j}, \overline{y}_{i,j}) + \min(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}},$$

$$\frac{\min(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\min(y_{j,i}, \overline{y}_{j,i})}{y_{j,i} + \overline{y}_{j,i}}),$$

$$\max(\frac{\max(y_{i,j}, \overline{y}_{i,j}) + \max(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}}, \frac{\max(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\max(y_{j,i}, \overline{y}_{j,i})}{y_{j,i} + \overline{y}_{j,i}})\rangle,$$

$$DJI^{av-opt}(F_i, F_j) = \langle \frac{\min(y_{i,j}, \overline{y}_{i,j}) + \min(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}},$$

$$\frac{\max(y_{i,j}, \overline{y}_{i,j}) + \max(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}}\rangle,$$

$$DJI^{av-pes}(F_i, F_j) = \langle \frac{\max(y_{i,j}, \overline{y}_{i,j}) + \max(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}},$$

$$\frac{\min(y_{i,j}, \overline{y}_{i,j}) + \min(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}}\rangle,$$

$$DJI^{pes}(F_i, F_j) = \langle \max(\frac{\max(y_{i,j}, \overline{y}_{i,j}) + \max(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}},$$

$$\frac{\max(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\max(y_{j,i}, \overline{y}_{j,i})}{y_{j,i} + \overline{y}_{j,i}}),$$

$$\min(\frac{\min(y_{i,j}, \overline{y}_{i,j}) + \min(y_{j,i}, \overline{y}_{j,i})}{y_{i,j} + \overline{y}_{i,j} + y_{j,i} + \overline{y}_{j,i}}, \frac{\min(y_{i,j}, \overline{y}_{i,j})}{y_{i,j} + \overline{y}_{i,j}}, \frac{\min(y_{j,i}, \overline{y}_{j,i})}{y_{j,i} + \overline{y}_{j,i}})\rangle,$$

Proposition 6. For every two factors F_i and F_j,
(a) their DJIs are IFPs,
(b) it is valid that

$$DJI^{opt}(F_i, F_j) \leq DJI^{av-opt}(F_i, F_j) \leq DJI^{av-pes}(F_i, F_j) \leq DJI^{pes}(F_i, F_j),$$

(c) they are valid equalities

$$DJI^{opt}(F_i, F_j) = CJI^{pes}(F_i, F_j),$$

$$DJI^{av-opt}(F_i, F_j) = CJI^{av-pes}(F_i, F_j),$$

$$DJI^{av-pes}(F_i, F_j) = CJI^{av-opt}(F_i, F_j),$$

$$DJI^{pes}(F_i, F_j) = CJI^{opt}(F_i, F_j).$$

Obviously, for every i, j:

$$DJI^{opt}(F_i, F_j) = \neg DJI^{pes}(F_i, F_j),$$

$$DJI^{av-opt}(F_i, F_j) = \neg DJI^{av-pes}(F_i, F_j).$$

The Consonance of the Joint Influence of factor F_i over all Other Factors (CJIOF), i.e., the factors from $\mathcal{F}^i$, has one of the three forms:

$$CJIOF^{opt}(F_i, \mathcal{F}^i) = \langle \max_{1 \le j \le n, j \ne i} y_{i,j}, \min_{1 \le j \le n, j \ne i} \overline{y}_{i,j} \rangle,$$

$$CJIOF^{av}(F_i, \mathcal{F}^i) = \langle \frac{1}{n-1} \sum_{1 \le j \le n, j \ne i} y_{i,j}, \frac{1}{n-1} \sum_{1 \le j \le n, j \ne i} \overline{y}_{i,j} \rangle,$$

$$CJIOF^{pes}(F_i, \mathcal{F}^i) = \langle \min_{1 \le j \le n, j \ne i} y_{i,j}, \max_{1 \le j \le n, j \ne i} \overline{y}_{i,j} \rangle.$$

Proposition 7. For every set $\mathcal{F}$ and for every i ($1 \le i \le n$),
(a) their CJIOFs are IFPs,
(b) it is valid that

$$CJIOF^{pes}(F_i, \mathcal{F}^i) \le CJIOF^{av}(F_i, \mathcal{F}^i) \le CJIOF^{opt}(F_i, \mathcal{F}^i).$$

The Dissonance of the Joint Influence of factor F_i over all Other Factors (DJIOF), i.e., the factors from $\mathcal{F}^i$, has one of the three forms:

$$DJIOF^{opt}(F_i, \mathcal{F}^i) = \langle \min_{1 \le j \le n, j \ne i} \overline{y}_{i,j}, \max_{1 \le j \le n, j \ne i} y_{i,j} \rangle.$$

$$DJIOF^{av}(F_i, \mathcal{F}^i) = \langle \frac{1}{n-1} \sum_{1 \le j \le n, j \ne i} \overline{y}_{i,j}, \frac{1}{n-1} \sum_{1 \le j \le n, j \ne i} y_{i,j} \rangle,$$

$$DJIOF^{pes}(F_i, \mathcal{F}^i) = \langle \max_{1 \le j \le n, j \ne i} \overline{y}_{i,j}, \min_{1 \le j \le n, j \ne i} y_{i,j} \rangle.$$

Proposition 8. For every set $\mathcal{F}$ and for every i ($1 \le i \le n$),
(a) their DJIOFs are IFPs,
(b) it is valid that

$$DJIOF^{opt}(F_i, \mathcal{F}^i) \le DJIOF^{av}(F_i, \mathcal{F}^i) \le DJIOF^{pes}(F_i, \mathcal{F}^i),$$

(c) they are valid equalities

$$DJIOF^{opt}(F_i, \mathcal{F}^i) = \neg CJIOF^{opt}(F_i, \mathcal{F}^i),$$

$$DJIOF^{av}(F_i, \mathcal{F}^i) = \neg CJIOF^{av}(F_i, \mathcal{F}^i),$$

$$DJIOF^{pes}(F_i, \mathcal{F}^i) = \neg CJIOF^{pes}(F_i, \mathcal{F}^i).$$

The Consonance of the Joint Influence of all Factors Excluding factor F_i (CJIFE), has one of the three forms:

$$CJIFE^{opt}(\mathcal{F}^i, F_i) = \langle \max_{1\leq j\leq n, j\neq i} y_{j,i}, \min_{1\leq j\leq n, j\neq i} \overline{y}_{j,i}\rangle,$$

$$CJIFE^{av}(\mathcal{F}^i, F_i) = \langle \frac{1}{n-1} \sum_{1\leq j\leq n, j\neq i} y_{j,i}, \frac{1}{n-1} \sum_{1\leq j\leq n, j\neq i} \overline{y}_{j,i}\rangle,$$

$$CJIFE^{pes}(\mathcal{F}^i, F_i) = \langle \min_{1\leq j\leq n, j\neq i} y_{j,i}, \max_{1\leq j\leq n, j\neq i} \overline{y}_{j,i}\rangle.$$

Proposition 9. For every set $\mathcal{F}$ and for every i $(1 \leq i \leq n)$,
(a) their CJIFEs are IFPs,
(b) it is valid that

$$CJIFE^{pes}(\mathcal{F}^i, F_i) \leq CJIFE^{av}(\mathcal{F}^i, F_i) \leq CJIFE^{opt}(\mathcal{F}^i, F_i).$$

The Dissonance of the Joint Influence of all Factors Excluding factor F_i (DJIFE), has one of the three forms:

$$DJIFE^{opt}(\mathcal{F}^i, F_i) = \langle \min_{1\leq j\leq n, j\neq i} \overline{y}_{j,i}, \max_{1\leq j\leq n, j\neq i} y_{j,i}\rangle.$$

$$DJIFE^{av}(\mathcal{F}^i, F_i) = \langle \frac{1}{n-1} \sum_{1\leq j\leq n, j\neq i} \overline{y}_{j,i}, \frac{1}{n-1} \sum_{1\leq j\leq n, j\neq i} y_{j,i}\rangle,$$

$$DJIFE^{pes}(\mathcal{F}^i, F_i) = \langle \max_{1\leq j\leq n, j\neq i} \overline{y}_{j,i}, \min_{1\leq j\leq n, j\neq i} y_{j,i}\rangle.$$

Proposition 10. For every set $\mathcal{F}$ and for every i $(1 \leq i \leq n)$,
(a) their DJIFEs are IFPs,
(b) it is valid that

$$DJIFE^{opt}(F_i, \mathcal{F}^i) \leq DJIFE^{av}(F_i, \mathcal{F}^i) \leq DJIFE^{pes}(F_i, \mathcal{F}^i),$$

(c) they are valid equalities

$$DJIFE^{opt}(\mathcal{F}^i, F_i) = \neg CJIFE^{opt}(\mathcal{F}^i, F_i),$$

$$DJIFE^{av}(\mathcal{F}^i, F_i) = \neg CJIFE^{av}(\mathcal{F}^i, F_i),$$

$$DJIFE^{pes}(\mathcal{F}^i, F_i) = \neg CJIFE^{pes}(\mathcal{F}^i, F_i).$$

The Influence of Factor F_i over all Other Factors (IFOFs), has one of the two forms:

$$IFOFs^{opt}(F_i, \mathcal{F}^i) = \langle \max_{1 \le j \le n, j \ne i} (y_{i,j}, \overline{y}_{i,j}), \min_{1 \le j \le n, j \ne i} (y_{i,j}, \overline{y}_{i,j}) \rangle,$$

$$IFOFs^{pes}(F_i, \mathcal{F}^i) = \langle \min_{1 \le j \le n, j \ne i} (y_{i,j}, \overline{y}_{i,j}), \max_{1 \le j \le n, j \ne i} (y_{i,j}, \overline{y}_{i,j}) \rangle,$$

Proposition 11. For every set $\mathcal{F}$ and for every i $(1 \le i \le n)$,
(a) their IFOFs are IFPs,
(b) it is valid that

$$IFOFs^{pes}(F_i, \mathcal{F}^i) \le IFOFs^{opt}(F_i, \mathcal{F}^i).$$

Obviously, for every i, j:

$$IFOFs^{opt}(F_i, \mathcal{F}^i) = \neg IFOFs^{pes}(F_i, \mathcal{F}^i).$$

The Influence of the Factors, different than factor F_i Over Factor F_i (IFsOF), has one of the two forms:

$$IFsOF^{opt}(\mathcal{F}^i, F_i) = \langle \max_{1 \le j \le n, j \ne i} (y_{j,i}, \overline{y}_{j,i}), \min_{1 \le j \le n, j \ne i} (y_{j,i}, \overline{y}_{j,i}) \rangle,$$

$$IFsOF^{pes}(\mathcal{F}^i, F_i) = \langle \min_{1 \le j \le n, j \ne i} (y_{j,i}, \overline{y}_{j,i}), \max_{1 \le j \le n, j \ne i} (y_{j,i}, \overline{y}_{j,i}) \rangle,$$

Proposition 12. For every set $\mathcal{F}$ and for every i $(1 \le i \le n)$,
(a) their IFsOFs are IFPs,
(b) it is valid that

$$IFsOF^{pes}(\mathcal{F}^i, F_i) \le IFsOF^{opt}(\mathcal{F}^i, F_i).$$

Obviously, for every i, j:

$$IFsOF^{opt}(F_i, \mathcal{F}^i) = \neg IFsOF^{pes}(F_i, \mathcal{F}^i).$$

The Consonance of the Joint Influence of factor F_i and all Other Factors ($CJI\&OF$), i.e., the factors from $\mathcal{F}^i$, has one of the three forms:

$$CJI\&OF^{opt}(F_i, \mathcal{F}^i) = \langle \max_{1 \le j \le n, j \ne i} (y_{i,j}, y_{j,i}), \min_{1 \le j \le n, j \ne i} (\overline{y}_{i,j}, \overline{y}_{j,i}) \rangle,$$

$$CJI\&OF^{av}(F_i, \mathcal{F}^i) = \langle \frac{1}{2(n-1)} \sum_{1 \le j \le n, j \ne i} (y_{i,j} + y_{j,i}),$$

$$\frac{1}{2(n-1)} \sum_{1 \le j \le n, j \ne i} (\overline{y}_{i,j} + \overline{y}_{j,i}) \rangle,$$

$$CJI\&OF^{pes}(F_i, \mathcal{F}^i) = \langle \min_{1 \le j \le n, j \ne i} (y_{i,j}, y_{j,i}), \max_{1 \le j \le n, j \ne i} (\overline{y}_{i,j}, \overline{y}_{j,i}) \rangle.$$

Proposition 13. For every set $\mathcal{F}$ and for every i $(1 \leq i \leq n)$,
(a) their CJI&OFs are IFPs,
(b) it is valid that

$$CJI\&OF^{pes}(F_i, \mathcal{F}^i) \leq CJI\&OF^{av}(F_i, \mathcal{F}^i) \leq CJI\&OF^{opt}(F_i, \mathcal{F}^i).$$

The Dissonance of the Joint Influence of factor F_i and all Other Factors ($DJI\&OF$), i.e., the factors from $\mathcal{F}^i$, has one of the three forms:

$$DJI\&OF^{opt}(F_i, \mathcal{F}^i) = \langle \min_{1 \leq j \leq n, j \neq i} (\overline{y}_{i,j}, \overline{y}_{j,i}), \max_{1 \leq j \leq n, j \neq i} (y_{i,j}, y_{j,i}) \rangle.$$

$$DJI\&OF^{av}(F_i, \mathcal{F}^i) = \langle \frac{1}{2(n-1)} \sum_{1 \leq j \leq n, j \neq i} (\overline{y}_{i,j} + \overline{y}_{j,i}),$$
$$\frac{1}{2(n-1)} \sum_{1 \leq j \leq n, j \neq i} (y_{i,j} + y_{j,i}) \rangle,$$

$$DJI\&OF^{pes}(F_i, \mathcal{F}^i) = \langle \max_{1 \leq j \leq n, j \neq i} (\overline{y}_{i,j}, \overline{y}_{j,i}), \min_{1 \leq j \leq n, j \neq i} (y_{i,j}, y_{j,i}) \rangle.$$

Proposition 14. For every set $\mathcal{F}$ and for every i $(1 \leq i \leq n)$,
(a) their DJI&OFs are IFPs,
(b) it is valid that

$$DJI\&OF^{opt}(F_i, \mathcal{F}^i) \leq DJI\&OF^{av}(F_i, \mathcal{F}^i) \leq DJI\&OF^{pes}(F_i, \mathcal{F}^i),$$

(c) they are valid equalities

$$DJI\&OF^{opt}(F_i, \mathcal{F}^i) = \neg CJI\&OF^{opt}(F_i, \mathcal{F}^i),$$
$$DJI\&OF^{av}(F_i, \mathcal{F}^i) = \neg CJI\&OF^{av}(F_i, \mathcal{F}^i),$$
$$DJI\&OF^{pes}(F_i, \mathcal{F}^i) = \neg CJI\&OF^{pes}(F_i, \mathcal{F}^i).$$

4. Conclusion

The correctness of each formula derived above is carefully proven. Additional connections in form of preference relations between the individual formulas in each group as well as among the formulas in different groups are given.

The proposed intuitionistic fuzzy interpretation of fuzzy cognitive maps allows the implementation of more precise tools for analysis of the systemic indicators and characteristics of socio-technical systems in different areas where the uncertainty introduced by complexity and human factor is very high. The preference relations between the intuitionistic fuzzy coefficients of consonance and dissonance could create an enhanced basis for informed decision making by ranking the influences and alternatives between the key factors.

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