Годишник на Секция "Информатика"
Съюз на учените в България
Том 3, 2010, 90-94

Annual of "Informatics" Section
Union of Scientists in Bulgaria
Volume 3, 2010, 90-94

On Lukasiewicz's intuitionistic fuzzy disjunction and conjunction

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1. Introduction

In [8] 10 different fuzzy implications are discussed. Having in mind that in the classical logic the equality

$$x \vee y = \neg x \to y, \tag{1}$$

where x and y are logical variables, $\vee$ - disjunction, $\to$ - implication and $\neg$ - negation, we see that for any implication we can construct a disjunction and after this, using De Morgan's laws - a conjunction. we will discuss a new form of a disjunction in the case of Intuitionistic Fuzzy Sets (IFSs; see [3]).

2. Definition and algebraic properties of Lukasiewicz's intuitionistic fuzzy disjunction and conjunction

The intuitionistic fuzzy propositional calculus has been introduced more than 20 years ago (see, e.g., [1, 3]). In it, if x is a variable then its truth-value is represented by the ordered couple

$$V(x) = \langle a, b \rangle,$$

so that $a, b, a + b \in [0, 1]$, where a and b are the degrees of validity and of non-validity of x and there the following definitions are given.

Below we shall assume that for the two variables x and y the equalities: $V(x) = \langle a, b \rangle, V(y) = \langle c, d \rangle$ $(a, b, c, d, a + b, c + d \in [0, 1])$ hold.

For two variables x and y operations "conjunction"(&), "disjunction"($\vee$), "implication"($\to$), and "(standard) negation"($\neg$) are defined by:

$$V(x \& y) = \langle \min(a, c), \max(b, d) \rangle,$$

$$V(x \vee y) = \langle \max(a, c), \min(b, d) \rangle.$$

In [4] the following two operations, which are analogues to operations "conjunction"and "disjunction are defined

$$V(x + y) = \langle a, b \rangle + \langle c, d \rangle = \langle a + c - ac, bd \rangle,$$

$$V(x.y) = \langle a, b \rangle . \langle c, d \rangle = \langle ac, b + d - bd \rangle.$$

The two standard modal operators (see [7]) have the following intuitionistic fuzzy estimations (see [2]).

$$V(\Box p) = \Box V(p) = \langle \mu(p), 1 - \mu(p) \rangle,$$

$$V(\Diamond p) = \Diamond V(p) = \langle 1 - \nu(p), \nu(p) \rangle.$$

During the last 20 years intuitionistic fuzzy predicative, intuitionistic fuzzy modal and intuitionistic fuzzy temporal logics were developed. They will be objects of future research.

Now, using (1) and intuitionistic fuzzy form of Lukasiewicz's implicarion (see [5, 6])

$$V(x \to_L y) = \langle a, b \rangle \to_L \langle c, d \rangle = \langle \min(1, b + c), \max(0, a + d - 1) \rangle,$$

we will introduce a disjunction with the following form of its estimation

$$V(x \vee_L y) = \langle a, b \rangle \vee_L \langle c, d \rangle = \langle \min(1, a + c), \max(0, b + d - 1) \rangle.$$

We will call the new disjunction *"Lukasiewicz's intuitionistic fuzzy disjunction"*.

We see also, that

$$V(x \to'_L y) = \neg \langle a, b \rangle \vee_L \langle c, d \rangle$$
$$= \langle b, a \rangle \vee_L \langle c, d \rangle = \langle \min(1, a + c), \max(0, b + d - 1) \rangle = V(x \to_L y),$$

i.e., the implication generates a disjunction that generates the initial implication.

In this first part of our research, we will suppose that De Morgan's laws are valid, i.e.,

$$x \& y = \neg(\neg x \vee \neg y). \tag{2}$$

We must note immediately, that in IFS theory there are a lot of examples in which (2) is not valid, but this will be object of discussions in future research.

Therefore, using (2) and definition of $\vee_L$, we can construct

$$V(x \wedge_L y) = \langle a, b \rangle \wedge_L \langle c, d \rangle = \langle \max(0, a + c - 1), \min(1, b + d) \rangle.$$

We will call the new conjunction *"Lukasiewicz's intuitionistic fuzzy conjunction"*.

For both new operations, having in mind that $\wedge_L$ is obtained from $\vee_L$ by (2), we will check fistly that

$$V(\neg(\neg x \wedge_L \neg y))$$
$$= \neg(\neg \langle a, b \rangle \wedge_L \neg \langle c, d \rangle))$$

$$= \neg(\langle b,a\rangle \wedge_L \langle d,c\rangle))$$

$$= \neg\langle \max(0, b+d-1), \min(1, a+c)\rangle$$

$$= \langle \min(1, a+c), \max(0, b+d-1)\rangle$$

$$V(x \vee_L y).$$

Therefore, both operations are correctly defined one about the other.

We can check immediately the validity of

Theorem 1 The following equalities are valid:

(a) $V(x \wedge_L y) = V(x \wedge_L y)$,

(b) $V(x \vee_L y) = V(x \vee_L y)$,

(c) $V((x \wedge_L y) \wedge_L z) = x \wedge_L (y \wedge_L z))$,

(d) $V((x \vee_L y) \vee_L z) = x \vee_L (y \vee_L z))$.

Proof. (d) Let x, y and z are three variables. Then

$$V((x \vee_L y) \vee_L z)$$

$$= (\langle a,b\rangle \vee_L \langle c,d\rangle) \vee_L \langle e,f\rangle$$

$$= \langle \min(1, a+c), \max(0, b+d-1)\rangle \vee_L \langle e,f\rangle$$

$$= \langle \min(1, \min(1, a+c)+e), \max(0, \max(0, b+d-1)+f-1)\rangle$$

$$= \langle \min(1, 1+e, a+c+e), \max(0, f-1, b+d+f-2)\rangle$$

$$= \langle \min(1, a+c+e), \max(0, b+d+f-2)\rangle$$

(because $1+e \geq 1$, $f-1 \leq 0$, $1+a \geq 1$ and $b-1 \leq 0$)

$$= \langle \min(1, 1+a, a+c+e), \max(0, b-1, b+d+f-2)\rangle$$

$$= \langle \min(1, a+\min(1, c+e)), \max(0, b+\max(0, d+f-1)-1)\rangle$$

$$= \langle a,b\rangle \vee_L \langle \min(1, c+e), \max(0, d+f-1)\rangle$$

$$= \langle a,b\rangle \vee_L (\langle c,d\rangle \vee_L \langle e,f\rangle)$$

$$= V(x \vee_L (y \vee_L z)).$$

In similar way we can prove the other equalities and

Theorem 2 Operations $\vee_L$ and $\wedge_L$ are distributive one in respect to the other.

Theorem 3 The following properties are valid:

(a) $(x \& y) \wedge_L z = (x \wedge_L z) \& (y \wedge_L z)$,

(b) $(x \vee y) \vee_L z = (x \vee_L z) \vee (y \vee_L z)$.

Theorem 4 The following properties are valid:

(a) $V(\Box (x \vee_L y)) = V(\Box x \vee_L \Box y)$,

(b) $V(\Box(x \wedge_L y)) = V(\Box x \wedge_L \Box y)$,

(c) $V(\Diamond(x \vee_L y)) = V(\Diamond x \vee_L \Diamond y)$,

(d) $V(\Diamond(x \wedge_L y)) = V(\Diamond x \wedge_L \Diamond y)$.

Proof. (a) Let x and y are two variables. Then

$$V(\Box(x \vee_L y)) = \langle a, b\rangle \vee_L \langle c, d\rangle$$

$$= \Box\langle \min(1, a+c), \max(0, b+d-1)\rangle$$

$$= \langle \min(1, a+c), 1 - \min(1, a+c)\rangle$$

$$= \langle \min(1, a+c), \max(0, 1-a-c)\rangle$$

$$= \langle \min(1, a+c), \max(0, (1-a)+(1-c)-1)\rangle$$

$$= \Box\langle a, 1-a\rangle \vee_L \langle c, 1-c\rangle$$

$$= V(\Box x \vee \Box y).$$

Acknowledgement

The first author is grateful for the support provided by the projects DID-02-29 “Modelling processes with fixed development rules"and BIn-2/09 “Design and development of intuitionistic fuzzy logic tools in information technologies"funded by the National Science Fund, Bulgarian Ministry of Education, Youth.

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