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**On Generalized Net Complexity**

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# 1 Introduction

Generalized Nets (GNs) are defined as extensions of the ordinary Petri nets and their modifications, but in a way that is principally different from the ways of defining the other types of Petri nets. The additional components in the GN-definition give more and larger modelling possibilities and determine the place of the GNs among the separate types of Petri nets, similar to the place of the Turing machine among the finite automata. On the other hand, the GN-definition is more complex than the definitions of the other Petri net modifications.

After studying of the publications on Petri nets from [8] it can be asserted that the first papers in which complexity operators are defined over some Petri net like objects are papers [2, 3, 4, 7]. In them some types complexity operators are defined over GNs. All these operators have only theoretical sense.

Here, we shall introduce new complexity operators that will give more adequate estimation for a given GN and by this reason they will have not only theoretical, but also practical applications.

In the beginning, we start with the definitions of the concepts "GN-transition"and "GN".

# 2 Short remarks on generalized nets

Following [5, 6], we shall introduce the concept of a GN-transition and of a Generalized Net (GN; for it see also [1, 9]).

Every GN-transition is described by a seven-tuple (Fig. 1):

$$Z = \langle L', L'', t_1, t_2, r, M, \square \rangle,$$

where:

**(a)** $L'$ and $L''$ are finite, non-empty sets of places (the transition's input and output places, respectively); for the transition in Fig. 3 these are $L' = \{l'_1, l'_2, \ldots, l'_m\}$ and $L'' = \{l''_1, l''_2, \ldots, l''_n\}$;

**(b)** $t_1$ is the current time-moment of the transition's firing;

**(c)** $t_2$ is the current value of the duration of its active state;

**(d)** $r$ is the transition's *condition* determining which tokens will pass (or *transfer*) from the transition's inputs to its outputs; it has the form of an Index Matrix (IM; see [46]):

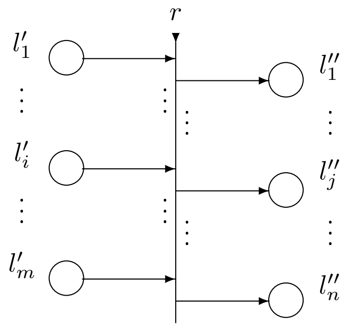

Fig. 1: GN-transition

$$r = \begin{array}{c|c} & l''_1 \ \dots \ l''_j \ \dots \ l''_n \\ \hline l'_1 & \\ \vdots & r_{i,j} \\ l'_i & (r_{i,j} - \text{ predicate }) \\ \vdots & (1 \le i \le m, 1 \le j \le n) \\ l'_m & \end{array};$$

$r_{i,j}$ is the predicate which corresponds to the $i$-th input and $j$-th output places. When its truth value is "$true$", a token from $i$-th input place can be transferred to $j$-th output place; otherwise, this is not possible;

**(e)** $M$ is an IM of the capacities of transition's arcs:

$$M = \begin{array}{c|c} & l''_1 \ \dots \ l''_j \ \dots \ l''_n \\ \hline l'_1 & \\ \vdots & m_{i,j} \\ l'_i & (m_{i,j} \ge 0 - \text{ natural number }) \\ \vdots & (1 \le i \le m, 1 \le j \le n) \\ l'_m & \end{array};$$

**(f)** $\square$ is an object having a form similar to a Boolean expression. It may contain as variables the symbols which serve as labels for transition's input places, and is an expression built up of variables and the Boolean connectives $\wedge$ and $\vee$ whose semantics is defined as follows:

| | | |
|---|---|---|
| $\wedge(l_{i_1}, l_{i_2}, \dots, l_{i_u})$ | - | every place $l_{i_1}$, $l_{i_2}$, $\dots$, $l_{i_u}$ must contain at least one token, |
| $\vee(l_{i_1}, l_{i_2}, \dots, l_{i_u})$ | - | there must be at least one token in all places $l_{i_1}$, $l_{i_2}$, $\dots$, $l_{i_u}$, where $\{l_{i_1}, l_{i_2}, \dots, l_{i_u}\} \subset L'$. |

When the value of a type (calculated as a Boolean expression) is "*true*", the transition can become active, otherwise it cannot.

The ordered four-tuple

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle,$$

$$\langle T, t^o, t^*\rangle, \langle X, \Phi, b\rangle\rangle$$

is called a *Generalized Net* (GN) if:

**(a)** $A$ is a set of transitions;

**(b)** $\pi_A$ is a function giving the priorities of the transitions, i.e., $\pi_A : A \rightarrow N$, where $N = \{0, 1, 2, \dots\} \cup \{\infty\}$;

**(c)** $\pi_L$ is a function giving the priorities of the places, i.e., $\pi_L : L \rightarrow N$, where $L = pr_1 A \cup pr_2 A$, and $pr_i X$ is the $i$-th projection of the $n$-dimensional set, where $n \in N, n \geq 1$ and $1 \leq k \leq n$ (obviously, $L$ is the set of all GN-places);

**(d)** $c$ is a function giving the capacities of the places, i.e., $c : L \rightarrow N$;

**(e)** $f$ is a function which calculates the truth values of the predicates of the transition's conditions (for the GN described here let the function $f$ have the value "*false*" or "*true*", i.e., a value from the set $\{0, 1\}$;

**(f)** $\theta_1$ is a function giving the next time-moment when a given transition $Z$ can be activated, i.e., $\theta_1(t) = t'$, where $pr_3 Z = t, t' \in [T, T + t^*]$ and $t \leq t'$. The value of this function is calculated at the moment when the transition terminates its functioning;

**(g)** $\theta_2$ is a function giving the duration of the active state of a given transition $Z$, i. e., $\theta_2(t) = t'$, where $pr_4 Z = t \in [T, T + t^*]$ and $t' \geq 0$. The value of this function is calculated at the moment when the transition starts functioning;

**(h)** $K$ is the set of the GN's tokens. In some cases, it is convenient to consider this set in the form

$$K = \bigcup_{l \in Q^I} K_l,$$

where $K_l$ is the set of tokens which enter the net from place $l$, and $Q^I$ is the set of all input places of the net;

**(i)** $\pi_K$ is a function giving the priorities of the tokens, i.e., $\pi_K : K \rightarrow N$;

**(j)** $\theta_K$ is a function giving the time-moment when a given token can enter the net, i.e., $\theta_K(\alpha) = t$, where $\alpha \in K$ and $t \in [T, T + t^*]$;

**(k)** $T$ is the time-moment when the GN starts functioning. This moment is determined with respect to a fixed (global) time-scale;

**(l)** $t^o$ is an elementary time-step, related to the fixed (global) time-scale;

**(m)** $t^*$ is the duration of the GN functioning;

**(n)** $X$ is the set of all initial characteristics the tokens can receive when they enter the net;

**(o)** $\Phi$ is a characteristic function which assigns new characteristics to every token when it makes a transfer from an input to an output place of a given transition.

**(p)** $b$ is a function giving the maximum number of characteristics a given token can receive, i.e., $b : K \rightarrow N$.

A GN may lack some of the components, and such GNs give rise to special classes of GNs called *reduced GNs*. The omitted elements of the reduced GNs are marked by "$*$".

## 3 New complexity operators over GNs

### 3.1 On the existing complexity operators over GNs

Let us denote by $\Phi$ the set of all complexity operators that we can define over a given GN $E$. The following operators described in [5], are examples for the elements of $\Phi$:

$$\varphi_1(E) = |pr_1pr_1E|, \varphi_1(E) = 0;$$

$$\varphi_2(E) = |pr_1pr_1pr_1E \cup pr_2pr_1pr_1E|, \varphi_2(E) = 0;$$

$$\varphi_3(E) = |pr_1pr_2E|, \varphi_3(E) = 0;$$

$$\varphi_4(E) = |pr_3pr_3E|, \varphi_4(E) = 0;$$

$$\varphi_5(E) = |pr_1pr_4E|, \varphi_5(E) = 0;$$

$$\varphi_6(E) = |pr_1pr_4E|, \varphi_6(E) = 0,$$

where $|X|$ is the cardinality of set $X$, $E$ is an empty GN, i.e., a GN without places, or transitions, or tokens, but in all cases – a GN that cannot functioning.

Some properties of the above operators are discussed in [5].

### 3.2 Statical complexity operator

The first of the new operators will be applied over a given GN-transition $Z$ with $m$ input and $n$ output places and wuth transition condition predicated $r_{i,j}$ for $1 \leq i \leq m, 1 \leq j \leq n$, and the result is a natural number corresponding to the number of all possible directions for tokens transfers. It has the form:

$$\kappa_s(Z) = \sum_{i=1}^{m} \sum_{j=1}^{n} f_s(r_{i,j}),$$

where

$$f_s(r_{i,j}) = \begin{cases} 1, & \text{if } r_{i,j} \neq false \\ 0, & \text{if } r_{i,j} = false \end{cases}$$

This operator can be extended over a whole GN $E$ and the new one will have the form

$$K_s(E) = \sum_{Z \in pr_1pr_1E} \kappa_s(Z).$$

### 3.3 Dynamical complexity operator

The second new operator is a modification of the first one. It also will be applied over a given GN-transition with the above form and the result is again a natural number corresponding to the number of all possible directions for tokens transfers, but now, only at the fixed time-moment $t$. This operator has the form:

$$\kappa_d(Z, t) = \sum_{i=1}^{m} \sum_{j=1}^{n} f_d(r_{i,j}, t),$$

where

$$f_d(r_{i,j}, t) = \begin{cases} 1, & \text{if at the current time-moment } t \ r_{i,j} = true \\ 0, & \text{otherwise} \end{cases}$$

This operator can be extended over a whole GN $E$ and the new one will have the form

$$K_d(E, t) = \sum_{Z \in pr_1 pr_1 E} \kappa_d(Z, t).$$

Obviously, for each time-moment $t \in [T, T + t^*]$

$$\kappa_d(Z, t) \leq \kappa_s(Z)$$

and

$$K_d(E, t) \leq K_s(Z).$$

### 3.4 Intuitionistic fuzzy complexity operators

First, we shall note that in intuitionistic fuzzy propositional and predicate logic to each variable $p$ two real numbers in interval $[0, 1]$ are assigned. They are called *"degree of truth (*or *validity)"* and *"degree of false (*or *non-validity)"* and they are marked by $\mu(p)$ and $\nu(p)$. For them the inequality $\mu(p) + \nu(p) \leq 1$ must hold. Let this assignment be provided by an evaluation function $V$ in such a way that: $V(p) = \langle \mu(p), \nu(p) \rangle$ that is called *intuitionistic fuzzy couple.*

When the values $V(p)$ and $V(q)$ of $p$ and $q$ are known, the evaluation function $V$ can also be extended to the operations "&″ and "$\vee$" as follows:

$$V(p\&q) = V(p)\&V(q) = \langle \min(\mu(p), \mu(q)), \max(\nu(p), \nu(q)) \rangle,$$

$$V(p \vee q) = V(p) \vee V(q) = \langle \max(\mu(p), \mu(q)), \min(\nu(p), \nu(q)) \rangle.$$

Another operation is "@". It is defined over variables $p_1, p_2, ..., p_n$:

$$\underset{i=1}{\overset{n}{@}} \ V(p_i) = \langle \frac{1}{n} \sum_{i=1}^{n} \mu(p_i), \frac{1}{n} \sum_{i=1}^{n} \nu(p_i) \rangle.$$

Now, we can introduce some new operators, that are modifications of the first one. They also will be applied over a given GN-transition with the above form, but now the result is an intuitionistic fuzzy couple of real numbers corresponding to the aggregated degrees of validity and of non-validity of the transition condition predicates of the given transition $Z$.

We will discuss only the statical form of these operator. For instance of the two above operators, the present ones have the following forms:
*optimistic operator:*

$$\iota_{opt}(Z) = \bigvee_{i=1}^{m} \ \bigvee_{j=1}^{n} \ V(r_{i,j}),$$

*average operator:*

$$\iota_{ave}(Z) = \underset{i=1}{\overset{m}{@}} \underset{j=1}{\overset{n}{@}} V(r_{i,j}),$$

*pesimistic operator:*

$$\iota_{opt}(Z,t) = \underset{i=1}{\overset{m}{\&}} \underset{j=1}{\overset{n}{\&}} V(r_{i,j}).$$

These operators can be extended over a whole GN $E$ and the new ones will have the forms

$$I_{opt}(E) = \sum_{Z \in pr_1 pr_1 E} \iota_{opt}(Z),$$

$$I_{ave}(E) = \sum_{Z \in pr_1 pr_1 E} \iota_{ave}(Z),$$

$$I_{pes}(E) = \sum_{Z \in pr_1 pr_1 E} \iota_{pes}(Z).$$

# 4 Properties of the new complexity operators

Let $E_1$ and $E_2$ be two GNs and let for $1 \leq i \leq 2$:

$$E_i = \langle\langle A_i, \pi_A^i, \pi_L^i, c^i, f^i, \theta_1^i, \theta_2^i\rangle, \langle K_i, \pi_K^i, \theta_K^i\rangle, \langle T_i, t_i^o, t_i^*\rangle, \langle X_i, \Phi_i, b_i\rangle\rangle$$

An operation *union* of two GNs is defined by:

$$E_1 \cup E_2 = \langle\langle A_1 \overline{\cup} A_2, \pi_A^1 \cup \pi_A^2, \pi_L^1 \cup \pi_L^2, c^1 \cup c^2, f^1 \cup f^2, \theta_1^1 \cup \theta_1^2,$$

$$\theta_2^1 \cup \theta_2^2\rangle, \langle K_1 \cup K_2, \pi_K^1 \cup \pi_K^2, \theta_K^1 \cup \theta_K^2\rangle,$$

$$\langle \min(T_1, T_2), GCD(t_1^o, t_2^o), \max_{1 \leq i \leq 2}(T_i + \frac{t_i^*.t_i^o}{GCD(t_1^o, t_2^o)} -$$

$$\min(T_1, T_2))\rangle, \langle X_1 \cup X_2, \Phi_1 \cup \Phi_2, b_1 \cup b_2\rangle\rangle$$

where

$$A_1 \overline{\cup} A_2 = \bigcup_{i=1}^{2} \{Z | (Z \in A_i) \& (\forall Z' \in A_{3-i})(Z \cap Z' = Z_\emptyset)\} \cup$$

$$\bigcup_{i=1}^{2} \{Z | (\exists Z' \in A_i)(\exists Z'' \in A_{3-i})(Z' \cap Z'' \neq Z_\emptyset) \& (Z = Z' \cup Z'')\}.$$

**Theorem** For each two GNs $E_1$ and $E_2$:
(a) $K_s(E_1 \cup E_2) \leq K_s(E_1) + K_s(E_2)$,

(b) $K_d(E_1 \cup E_2) \leq K_d(E_1) + K_d(E_2)$,

(c) $I_{opt}(E_1 \cup E_2) \leq I_{opt}(E_1) + I_{opt}(E_2)$,

(d) $I_{ave}(E_1 \cup E_2) \leq I_{ave}(E_1) + I_{ave}(E_2)$,

(e) $I_{pes}(E_1 \cup E_2) \leq I_{pes}(E_1) + I_{pes}(E_2)$.

The validity of these properties follow from the above definitions.

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