Годишник на секция "Информатика"
Съюз на учените в България
Том 2, 2009, 29-30

Annual of "Informatics" Section
Union of Scientists in Bulgaria
Volume 2, 2009, 29-30

Remark on Equalities between Intuitionistic Fuzzy Sets

Krassimir T. Atanassov
CLBME-Bulgarian Academy of Sciences, P.O. Box 12, Sofia-1113, Bulgaria
krat@bas.bg

In [1, 2], the following equality is proved for two fixed Intuitionistic Fuzzy Sets (IFSs, see [1]) A and B:

$$((A \cap B) + (A \cup B))@((A \cap B).(A \cup B)) = A@B,$$

where operations "$\cap$", "$\cup$", "+", "." and "@" are defined by

$$\begin{aligned}
A \cap B &= \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle | x \in E\},\\
A \cup B &= \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle | x \in E\},\\
A + B &= \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x), \nu_A(x).\nu_B(x)\rangle \mid x \in E\},\\
A.B &= \{\langle x, \mu_A(x).\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x).\nu_B(x)\rangle \mid x \in E\},\\
A@B &= \{\langle x, (\frac{\mu_A(x) + \mu_B(x))}{2}, \frac{(\nu_A(x) + \nu_B(x))}{2}\rangle | x \in E\}.
\end{aligned}$$

Here, we shall formulate and prove four other elementary, but interesting equalities.
Theorem 1: For every two IFSs A and B:

$$(A \cap B)@(A \cup B)) = (A + B)@(A.B).$$

Proof: We shall use the fact that for every two real numbers a and b it follows

$$\max(a, b) + \min(a, b) = a + b.$$

Then

$$\begin{aligned}
&(A \cap B)@(A \cup B))\\
= \;& (\{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle | x \in E\}\\
&@\{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle | x \in E\})
\end{aligned}$$

$$
\begin{aligned}
&= \{\langle x, \frac{\min(\mu_A(x),\mu_B(x)),\max(\mu_A(x),\mu_B(x))}{2}, \\
&\frac{\max(\nu_A(x),\nu_B(x))+\min(\nu_A(x),\nu_B(x))}{2}\rangle | x \in E\}) \\
&= \{\langle x, \frac{\mu_A(x)+\mu_B(x)}{2}, \frac{\nu_A(x)+\nu_B(x)}{2}\rangle | x \in E\})
\end{aligned}
$$

$$
\begin{aligned}
&= \{\langle x, \frac{(\mu_A(x)+\mu_B(x)-\mu_A(x).\mu_B(x))+\mu_A(x).\mu_B(x)}{2}, \\
&\frac{\nu_A(x).\nu_B(x)+(\nu_A(x)+\nu_B(x)-\nu_A(x).\nu_B(x))}{2}\rangle | x \in E\} \\
&= \{\langle x, \mu_A(x)+\mu_B(x)-\mu_A(x).\mu_B(x), \nu_A(x).\nu_B(x)\rangle | x \in E\} \\
&@\{\langle x, \mu_A(x).\mu_B(x), \nu_A(x)+\nu_B(x)-\nu_A(x).\nu_B(x)\rangle | x \in E\} \\
&= (A+B)@(A.B).
\end{aligned}
$$

The proofs of the next assertions are analogous.

Theorem 2: For every two IFSs A and B:

$$\neg(A \cap B) \rightarrow (A \cap B) = (A \rightarrow B) \cup (B \rightarrow A),$$

where operations "$\neg$"and "$\rightarrow$"are defined by

$$\neg A = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\},$$

$$A \rightarrow B = \{\langle x, max(\nu_A(x), \mu_B(x)), min(\mu_A(x), \nu_B(x))\rangle | x \in E\}.$$

They are the standard intuitionistic fuzzy negation and implication that are extensions of Kleene-Dienes negation and implicaion (see [3]).

Theorem 3: For every three IFSs A, B and C:

$$(A \rightarrow B) \cup (B \rightarrow C) \cup (C \rightarrow A) = (A \rightarrow C) \cup (C \rightarrow B) \cup (B \rightarrow A),$$

$$(A \rightarrow B) \cap (B \rightarrow C) \cap (C \rightarrow A) = (A \rightarrow C) \cap (C \rightarrow B) \cap (B \rightarrow A).$$

At the moment an **open problem** is whether there are equalities similar the the last one, but for the other intuitionistic fuzzy implications.

References

[1] Atanassov K., An equality between intuitionistic fuzzy sets, Fuzzy sets and Systems Vol. 79 (1996), No. 2, 257-258.

[2] Atanassov K., Intuitionistic Fuzzy Sets, Springer Physica-Verlag, Heidelberg, 1999.

[3] Klir, G. and Bo Yuan, *Fuzzy Sets and Fuzzy Logic.* Prentice Hall, New Jersey, 1995.