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On intuitionistic fuzzy subtractions $-'_{20}$ and $-''_{20}$

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Abstract: An intuitionistic fuzzy subtraction, generated by the intuitionistic fuzzy negation $\neg_{20}$ is constructed. Some of its basic properties are studied.

1 Introduction

In a series of papers, more than 150 different intuitionistic fuzzy implications were defined by K. Atanassov, D. Dimitrov, B. Kolev, T. Trifonov and others.

As it was discussed in [1, 2], in intuitionistic fuzzy propositional calculus, if x is a variable then its truth-value is represented by the ordered couple

$$V(x) = \langle a, b \rangle,$$

so that $a, b, a + b \in [0, 1]$, where a and b are degrees of validity and of non-validity of x.

Below we shall assume that for the three variables x, y and z the equalities: $V(x) = \langle a, b\rangle, V(y) = \langle c, d\rangle, V(z) = \langle e, f\rangle$ $(a, b, c, d, e, f, a+b, c+d, e+f \in [0, 1])$ hold.

For two variables x and y the operation "conjunction"(&) is defined (see [1]) by:

$$V(x\&y) = \langle \min(a, c), \max(b, d)\rangle.$$

2 Main results

In [3] different intuitionistic fuzzy forms of operation "subtraction"are introduced. They are based on the intuitionistic fuzzy forms of operation "negation". One of these negations - $\neg_{20}$ was introduced by the author in [4, 5]. It has the form

$$\neg_{20}\langle a, b\rangle = \langle b, 0\rangle.$$

Now, we will transform these results to the case of Intuitionistic Fuzzy Sets (IFSs, see [2]). Let the IFSs

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$$

and

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in E\}$$

be given. Then, we can define for them:

$$\neg_{20} A = \{\langle x, \nu_A(x), 0\rangle | x \in E\}.$$

Therefore,

$$\neg_{20}\neg_{20} A = \{\langle x, 0, 0\rangle | x \in E\}.$$

Now, operation "subtraction" will have two different forms:

$$A -'_{20} B = \{\langle x, \min(\mu_A(x), \nu_B(x)), \nu_A(x)\rangle | x \in E\}$$

and

$$A -''_{20} B = \{\langle x, 0, \nu_A(x)\rangle | x \in E\}.$$

We can check that as a result of the operations "$-'_{20}$" and "$-''_{20}$" we obtain IFSs. Really, for two given IFSs A and B and for each $x \in E$ we obtain that

$$0 \leq \min(\mu_A(x), \nu_B(x)) + \nu_A(x) \leq \mu_A(x) + \nu_A(x) \leq 1.$$

The check for the second subtraction is similar.

Let us make use of the *empty IFS*, the *totally uncertain IFS*, and the *unit IFS* (as defined in [2]) by:

$$O^* = \{\langle x, 0, 1\rangle | x \in E\},$$

$$U^* = \{\langle x, 0, 0\rangle | x \in E\},$$

$$E^* = \{\langle x, 1, 0\rangle | x \in E\}.$$

Let

$$A^* = \{\langle x, 0, \nu_A(x)\rangle | x \in E\}$$

The validity of the following assertions is directly checked

Theorem 1: For every IFS A:

(a) $A -'_{20} E^* = A^*$,

(b) $A -'_{20} O^* = A$,

(c) $A -'_{20} U^* = A^*$,

(d) $E^* -'_{20} A = \neg_{20} A$,

(e) $O^* -'_{20} A = O^*$,

(f) $U^* -'_{20} A = U^*$.

Theorem 2: For every two IFSs A and B:

(a) $(A -'_{20} B) \cap C = (A \cap C) -'_{20} B = A \cap (C -'_{20} B)$,

(b) $(A \cap B) -'_{20} C = (A -'_{20} C) \cap (B -'_{20} C)$,
(c) $(A -'_{20} B) -'_{20} C = (A -'_{20} C) -'_{20} B$.

Theorem 3: The following equalities hold:
(a) $O^* -'_{20} U^* = O^*$,
(b) $O^* -'_{20} E^* = O^*$,
(c) $O^* -'_{20} O^* = O^*$,
(d) $U^* -'_{20} U^* = U^*$,
(e) $U^* -'_{20} E^* = U^*$,
(f) $U^* -'_{20} O^* = U^*$,
(g) $E^* -'_{20} U^* = U^*$,
(h) $E^* -'_{20} E^* = U^*$,
(i) $E^* -'_{20} O^* = U^*$.

Theorem 4: For every two IFSs A and B:

$$A -''_{20} B = A^*.$$

Theorem 5: For every IFS A:
(d) $E^* -''_{20} A = U^*$,
(e) $O^* -'_{20} A = O^*$,
(f) $U^* -'_{20} A = U^*$.

Theorem 6: For every two IFSs A and B:
(a) $(A -''_{20} B) \cap C = (A \cap C) -''_{20} B = A \cap (C -''_{20} B)$,
(b) $(A \cap B) -''_{20} C = (A -''_{20} C) \cap (B -''_{20} C)$,
(c) $(A -''_{20} B) -''_{20} C = (A -''_{20} C) -''_{20} B$.

Theorem 7: The following equalities hold:
(a) $O^* -''_{20} U^* = O^*$,
(b) $O^* -''_{20} E^* = O^*$,
(c) $O^* -''_{20} O^* = O^*$,
(d) $U^* -''_{20} U^* = U^*$,
(e) $U^* -''_{20} E^* = U^*$,
(f) $U^* -''_{20} O^* = U^*$,
(g) $E^* -''_{20} U^* = U^*$,
(h) $E^* -''_{20} E^* = U^*$,
(i) $E^* -''_{20} O^* = E^*$.

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