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On Mathematics and Informatics as Sciences

Dimitar Dobrev Dobrev
Sofia University "St. Kliment Ohridski", Faculty of Mathematics and Informatics
dobrev@fmi.uni-sofia.bg

Abstract: Without going into a comparative analysis of the existing variety of definitions of mathematics and informatics as sciences, we propose two new and original definitions of these concepts. We discuss the motives for proposing these definitions and the basic claims of these formulations. We clarify the conceptual context of these definitions and, specifically, the essence of such fundamental philosophical concepts as *object, subject* (active and capable subject)*, model, modeling, knowledge*, etc. In conclusion we attempt to formulate some joint problems of mathematics and informatics, problems that are topical and significant at the present stage of development and application of these two sciences.

1. Motives

The invention of the computer set the beginning of a new and important orientation in science. The term *computer science* only marks this important scientific and applied field, but unfortunately does little to clarify its fundamental essence and its organic connection with mathematics.

In the course of the vigorous development of informatics such a terminological vagueness can lead to negative phenomena. For instance:

- The general scientific terms *object* and *modeling* unrightfully appropriated by informatics and mathematics respectively for their private use, opened an undesirable terminological niche that can hardly be filled in an adequate way.
- The term *cybernetics*, fetichized and spontaneously overused in the 1960s, eventually came to be devalued, which is understandable, and in fact disappeared.
- The term *information*, widely used as a synonym of *data*, today no longer has any real chance of being semantically generalized so as to embrace other kinds of information, for instance those we associate with the notion of *skills*, or *objectives*, or *postulate*, or *criteria*.
- The widespread dissemination of the programme language C++, a result of a functionally successful yet planless terminological development, has harbored in its lexical stock a strange mixture of all sorts of *functions, switches, templates, friends*, *import, export, inheritance*, etc, thus making impossible the streamlined presentation of its own model paradigm.
- In informatics there is a definite lack of a unifying term for denoting the many kinds of specific cases of active dynamic systems with a selective behavior based on different kinds of information.

- We should explicitly point out that this last point also applies to Man as an actual active component of various homogenous or heterogeneous systems displaying an informational behavior.

As a first step to resolving this inadequacy, and without going into a comparative analysis of the existing variety of definitions of mathematics and informatics as sciences, we here propose two new and original definitions of these concepts.

Specifying their terminological context is the second necessary step in this direction. Even the first steps are already promising for a more productive further development.

2. Basic Claims of Definitions

The classical means for objectivizing fundamental scientific knowledge are concepts. Terms and definitions for their part are a form of notation and semantic concretization of concepts. As such, and in order for them to be really effective and productive, terms and definitions should meet a number of important requirements:

- Definitions should be clear, categorical and exhaustive both with regard to the essence of their object and with regard to its natural conceptual and functional surroundings.
- Definitions, together with their conceptual context, should form a consistent system adequately reflecting the objective properties of the respective topical sphere.
- When referring to other terms, definitions must stick to the basic semantics of the latter, and should explicitly specify objective particularities when there are such.
- Definitions must harmoniously and fully combine in unity the theoretical and practical manifestations of their object.
- Definitions must seek a good balance between generality and concreteness.
  The excessive shrinking of the conceptual framework is an obstacle to the natural development in the theoretical and applied aspect.
  The excessive extension of conceptual frameworks allows semantic generalizations, but may also lead to dissolving borderlines to the point where the concept loses its essence.
- Definitions should be accessible and clear for the widest possible circle of users.
- It is not desirable to introduce new terms, or for their definitions to be in harsh contrast with the usual semantics of the national or international vocabulary that has come to be widely used.

3. Two Main Concept Definitions

Relying on adequate general notions about the essence of ***mathematics*** and ***informatics***, we will go on directly to the two new definitions proposed:

> ***Mathematics is a science that formulates the principles of,***
> ***creates, studies and applies the formal means and methods of,***
> ***the analysis, synthesis and use of***
> ***abstract model systems.***

Informatics is a science that formulates the principles,
creates, studies and applies the formal means and methods of,
the analysis, synthesis, behavior, management, and interaction of/with
object, subject and environmental systems
(natural or artificial: applied, abstract, model or virtual systems)
based on, and/or operating with formalized knowledge
(postulates, factologicalual, procedural, conceptual knowledge, and meta-knowledge).

4. Argumentation

Although the preset goal of this investigation was to clarify the essence of informatics, we decided a feasible and more easily achievable intermediate goal was to construct a definition of the concept of mathematics.

The obvious similarity in the structure and contents of the two definitions is not a result of some a priori bias, but reflects the objective conceptual similarities between these two sciences.

We consciously sought the level of generality of these two definitions. This level was achieved through a kind of "mini-max" procedure where every word was put to critical analysis in order to assess its expediency according to the criterion "What would the definition gain or lose if this word were added or removed?"

It is imperative, however, that we explain the complete absence of the term *information*. As already noted, due to a long history of one-sided use going back to the early times of computers, the term has lost much of its qualitative essence and at present is hardly suitable for the wide range of semantic roles that it is expected to encompass as a concept. As an appropriate substitute we use the semantically richer term *knowledge*.

The term *computer* is also completely absent. We could discern its unobvious presence behind the term *formal means* if we imagined them in their engineering materialization.

The most significant difference between the two definitions is the presence in one and the absence in the other of the notion of *environment*. An unobvious corollary of this is the fact that the only active material environment for model processes natural to mathematics is Man, who has consciously reduced his psychical resources to those of the formal model systems created by him. For informatics the material environment is the actual technical systems constructed to work according to principles originating from this science. That is why, compared with mathematical environments, computer environments possess at least one additional level of complexity internal to the model. Another important result of the inclusion of computers in model processes is that the two types of Man-machine interface (that of programming and that of the consumer) become an object of analysis and formalization.

The absence of the term *computer*, however, is not a coincidence. It is connected with the attempt to achieve in the definition of informatics, a level of generality that would make it possible, if necessary, to include Man as an information subject of equal rights in a heterogeneous system with informational behavior – and there is urgent need for this inclusion when investigating and mastering the contemporary anthropotechnical symbiosis.

5. The Conceptual Context

Explicitly or tacitly given keywords connected with the two definitions proposed, besides the word science, are such commonly understandable words as object and system, means and methods, analysis and synthesis, theory and practice. However the conceptual essence of the terms *formality, modeling*, and *knowledge* is not clear enough and hence refers to such derivative concepts as *model* and *virtual environments*.

5.1. On the Formal Nature of Mathematics and Informatics

The property of *formality* is inherent, in principle and a priori, to the means (environments, objects and transformations) as well as the respective methods used in mathematics and informatics. Attributive properties and manifestations of formality are:

Materializability - the possibility for the means and methods to be presented and to function in material substances having optic, acoustic and other properties, usually in the form of discrete sign notations with a suitable structure, and, less often, also as systems of elements with a spatial and/or temporal continuity.
The material substance used, the concrete way of presenting and the effect of the means and methods are technologically important, but not of major significance.
In this sense, and for greater ease, their choice can vary in quite a wide range.

Abstractness - achieving a level of generality where the only remaining significant characteristics are fundamental and essential ones such as: static and dynamic, discrete and continuous; as well as properties and relations between them such as: quantitative, spatial, temporal, structural, genealogical ones, cause and effect relations, behavioral, etc.
Axiomatical characteristic - following predefined clear and rigorous semantics of the formal means and methods, semantics that exclude any element of arbitrariness in the realization and interpretation of their properties.
This rigor does not at all preclude the possibility of postulating changeability - evolutionary or even revolutionary – including the phenomena of primal genesis and complete destruction.

Validity - the requirement that every concrete formal system, with its composition, structure and properties as set down by definition, must not be internally contradictory.

Analytical characteristic - this property is a corollary and manifestation of the above-listed properties and consists in the suitability of such systems for use as a reliable formal substance for synthesis, but likewise as a convenient instrumental environment for analyzing various production system, one that guarantees the possibility of establishing their properties and predicting their behavior.

5.2. On the Model Essence of Mathematics and Informatics

A key role for clarifying the model essence of mathematics and informatics as sciences, is played by the concepts of *subject, knowledge, modeling*, viewed in their organic interconnectedness.

An Important "Lemma"

In seeking maximum generality, and limiting ourselves only to the most essential, we propose the following abstract definition of the concept of subject:

The subject is an open dynamic system, possessing functionally specialized components (organs): Senses, effectors and memory, which have the following functional properties and manifestations:

Sensibility – with regard to external and internal events.

Effectors – external or internal to the subject.

Reflex – the presence of internal effectors, conditioned by sensibility and having a permanent impact on memory.

These three properties united make possible the active behavior of the subject, including the following properties of behavior:

Inductivity – initial excitability by external stimuli, evident in non-voluntary, i.e. unconditioned, internal effector response, (possibly combined with reflex and/or external effector response).

Activeness – the capacity of the stimulated subject to perform activities one after the other according to an a priori, and possibly reflexly conditioned, scheme of behavior (possibly with the temporal parallel of the activities being carried out).

Selectivity – a priori and/or reflexly conditioned selectivity of behavior, directly or indirectly attainable for every form of activity with the exception of the inductive.

The proposed abstract definition claims to be equally well suited to the actually existent biological subjects and to all kinds of known anthropogenic and technogenic subjects, such as computers, and to their abstract formal analogues automata, well known to us from discrete mathematics.

5.3. The Principle of Aggregation of Subjects

An important corollary of the above definition is the fact that when there is effector-sensory compatibility, two or more subjects can come into direct, and even indirect, interaction. Under conditions of a priori and/or conditioned similarity of the schemes of behavior and of the criteria of selectivity, the subjects interacting thereby can realize even more complicated schemes of collective behavior based on the principles of specialization and division of activities. This is true both for homogeneous and heterogeneous configurations and populations of subjects, including those in which people take part. This is the reason why such systems of subjects are viewed as macro-subjects and why we claim that the exceptionally important principle of aggregation is generally applicable to all possible varieties of subjects.

5.4. Model-based Behavior

The *model* is a system (a model system) which the subjects chose in the framework of its postulated and/or empirically established by them similarity with another chosen system (object system); the subjects use it in its quality of image (analogue, model) of that other system in order to achieve higher effectiveness in the analysis, synthesis, management of and interaction with the respective object system.

Such an approach of the subjects in their interaction with the surrounding world (natural, abstract, model, virtual or hybrid world), is called a *model approach*.

The model systems created through the model approach possess the properties of real or virtual objects in the world surrounding the subjects that have created them.

The useful effect of the model approach is based on the possibility for mental abstraction (to the necessary degree) from the concrete aspects of the object system in its material, energy, spatial, temporal, behavioral, quantitative and all other manifestations.

The useful effect of the model approach consists in the convenience, for the subject, of the facilitated access to the components of the model system and the better knowledge of their properties and behavior, as well as the ampler and more convenient possibilities for manipulating them.

Basic stages and problems connected with applying the model approach are:

- Discovering an existing or constructing an artificial, system, which could fulfil the role of a model system with respect to the object system that is of theoretic or practical interest for the subject.
- Interpreting the object system in terms of the model system.
- Establishing the limits of the similarity between the two systems (object and model).
- Manipulations in the model system in order to purse the goals of the subject with regard to the object system.
- Interpreting these results in the terms of the object system.
- Verifying the results obtained in the model system in order to remove the objective differences between the object system and the model system.

A fact of major importance is that the “physiology” of a subject (real, abstract or formal) proves to be compatible with the model approach. Moreover, careful analysis shows that:

- Due to the reflex basis of his/her behavior, every subject in fact applies the model approach when interacting with the external environment and with him/herself.
- For a potential external respondent, a subject could fulfill the role of a kind of model system, reflexively adapted to the changes in the outside world – of course, only in the framework of his/her own internal structure and complexity.
- As an element of an organized social system, a subject can assume the role of a functionally specialized active bearer of one or several kinds of distributed knowledge, and thus likewise assume the role of an active component of some virtually distributed model environment.

In cases when the newly created model systems, for their part, also possess the properties of subjects, the existent ones and the newly created can continue to exist in relationships of active interaction. A necessary condition for this is the interface compatibility of subjects. Under the conditions of a priori and/or reflexively acquired similarity of the respective schemes of behavior and of the criteria of selectivity, such homogeneous or heterogeneous co-societies and populations can in general achieve considerably more complicated schemes of behavior.

Being based on the formula

$$\textit{specialization} \rightarrow \textit{division of labor} \rightarrow \textit{integration}$$

they come close in their characteristics to the behavior schemes of natural social systems. A direct corollary, fundamental for theory and practice, is the following principle of aggregation:

Homogeneous and heterogeneous systems and populations of subjects, under conditions of appropriate specialization and labor division, can acquire the properties and display the behavior of virtual macro-subjects.

5.5. Model Environments

The model environment is an instrumental environment for model activity.

Hence, functionally complete formal systems of instrumental means of expression (factological and procedural), which provide for and support the realization of an integral complex of activities, connected with applying the model approach, based on a model conception (paradigm), defined a priori and oriented to a certain class of problems from a chosen topical field.

5.6. Virtual Environments

The virtual environment is the life environment of natural and/or formal subjects.

Hence, a functionally complete system of materialized formal objects and subjects, whose outward manifestations and behavior are compatible with the outward manifestations and behavior of existing subjects or subject systems (natural or formal), and which, in this capacity, fulfill the role of an appropriately organized life environment for those subjects. Once constructed, materialized and activated, model and virtual environments become real environments for their users and respondents.

6. On the Behavior Role of Knowledge

Clarifying the essence of the concept of *knowledge* is a very complicated philosophical problem, which is not part of our topic.

We will limit ourselves to formulating some requirements concerning knowledge, namely:

That it be of proper quality (i.e. valid, reliable, as full as possible),
and formalized (in the sense discussed above).

The functional role of various kinds of knowledge will be significant here. In terms of the concepts of informatics, we distinguish five basic kinds of knowledge. We can identify them using some appropriate questions as a touchstone:

Postulative knowledge, associated with the question “On what grounds?”

Factgrafical knowledge, associated with the questions “Who, where, when, what, how much?”

Procedural knowledge, associated with the question “How?”

Conceptual knowledge, associated with the question “Why?”

Meta-knowledge, associated with general questions:
about the relationship of the knowledge with the modeled and modeling system,
about the environment in which this knowledge functions,
about the role of subjects as creators, bearers, and users of this knowledge,
and about the relationship of such knowledge to other, similar knowledge.

We should explicitly point out that the borderline separating factological from procedural knowledge is not always so definite. For instance factology implemented in a reference table, interpreted through appropriate procedure for selective search, actually becomes a kind of procedural knowledge that carries out a discrete transformation.

The use of a suitable notation somewhat neutralizes such an ambiguity, but not entirely. Thus the familiar formula of Pythagoras, $\mathbf{a^2 + b^2 = c^2}$, expresses a generally valid, formalized, factological knowledge about the sides of any right-angled triangle. In the form $\mathbf{c = |\, (a^2 + b^2)^{1/2}\, |}$, the formula represents a concrete procedural knowledge about a similar geometric object. But considered as objects of storage, transport or visualization, the two formulas are no more than various superscript linear texts.

Functional ambiguity can be also observed to some degree between conceptual knowledge and meta-knowledge.

7. Formal Subject Variety

Such generalized notions about the essence of informatics and the notions connected with them, allow us to deduce some more useful results as corollaries. We mean the new unifying view of the different kinds of active dynamic systems with selective behavior based on knowledge. The missing link can be found in the generalized concept, formulated above, of an abstract subject. Thanks to this notion, it is actually possible to reduce to a common denominator well-known systems like neurons, automata, functions, procedures, objects, and agents, together with neuron networks, cell automata, recursive functions, hierarchies of procedures, multi-agent systems, and, to the same degree, all sorts of one-processor and multi-processor computer architectures, as well as computer and communication networks. We should only add that Man himself and all the possible coalitions of people, in their capacity of virtual subjects, fall into the same category.

8. Joint Theoretical and Applied Topics

The training of the present-day generation of specialists in mathematics and informatics, and the level of their knowledge about the fundamental role of their sciences, allows us to formulate and address in a great measure the following topical joint tasks:

1. Impermeating the everyday environment of modern people with integrated environments and technologies that enhance the culture of their life with regard to information.
2. Developing integrated professional environments and technologies, which permit the effective realization of the entire process:

- ranging from the formulation of a class of tasks connected with the respective topical area,
- to the synthesis of their adequate modeling
- and to their full practical exploitation.

All this, at the level and in terms of the subjects' knowledge about the respective specific professional field.

3. Enriching and structuring mathematics and informatics (conceptually, theoretically, and technologically) to the level of a valuable foundation for solving the above-mentioned two tasks in various new concrete topical fields.
4. Elaborating the principles, methods, means and environments, suitable for producing different kinds of formalized knowledge in concrete or wider topical fields.

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