

GENERALIZED NETS HAVING PLACES WITH LIMITED GLOBAL CAPACITIES

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A variety of different types of Generalized Net (GN) extensions have been defined and each of them has been proven to be a conservative extension of the ordinary GNs (see e.g. [1,2]).

In this note, we introduce yet another extension and prove that it is a conservative one.

We use the following notations throughout:

- $\mathcal{N} = \{0, 1, 2, \dots\} \cup \{\infty\}$;
- $pr_i X$ is the i -th projection of the n -dimensional set, where $n \in \mathcal{N}$, $n \geq 1$, and $1 \leq i \leq n$. More generally, for a given n -dimensional set X ($n \geq 2$)

$$pr_{i_1, i_2, \dots, i_k} X = \prod_{j=1}^k pr_{i_j} X$$

where $1 \leq i_1 < i_2 < \dots < i_k \leq n$.

- $card(X)$ is the cardinality of set X .

As for the GN-specific notation, we refer the reader to [1,2].

The formal definition of the new type of extension coincide with this of the ordinary GN. Every transition is described by a seven-tuple (see Fig. 1):

$$Z = \langle L', L'', t_1, t_2, r, M, \square \rangle,$$

where:

(a) L' and L'' are finite, non-empty sets of places (the transition's input and output places, respectively); for the transition in Fig. 1 these are

$$L' = \{l'_1, l'_2, \dots, l'_m\}$$

and

$$L'' = \{l''_1, l''_2, \dots, l''_n\};$$

(b) t_1 is the current time-moment of the transition's firing;

(c) t_2 is the current value of the duration of its active state;

(d) r is the transition's *condition* determining which tokens will transfer from the transition's inputs to its outputs. Parameter r has the form of an Index Matrix (IM, see [1,2]):

$$r = \begin{array}{c|ccc} & l''_1 & \dots & l''_j & \dots & l''_n \\ \hline l'_1 & & & & & \\ \vdots & & & & & \\ l'_i & (r_{i,j} & - & \text{predicate}) & & \\ \vdots & & & & & \\ l'_m & (1 \leq i \leq m, 1 \leq j \leq n) & & & & \end{array} ;$$

where $r_{i,j}$ is the predicate which expresses the condition for transfer from the i -th input place to the j -th output place with the obvious meaning: whenever $r_{i,j}$ has truth-value "true", a token from the i -th input place can be transferred to the j -th output place; otherwise, such a transfer is not allowed;

(e) M is an IM of the capacities of transition's arcs:

$$M = \begin{array}{c|ccc} & l''_1 & \dots & l''_j & \dots & l''_n \\ \hline l'_1 & & & & & \\ \vdots & & & & & \\ l'_i & (m_{i,j} \geq 0 & - & \text{natural number or } \infty) & & \\ \vdots & & & & & \\ l'_m & (1 \leq i \leq m, 1 \leq j \leq n) & & & & \end{array} ;$$

(f) \square is called transition type and it is an object having a form similar to a Boolean expression. It may contain as variables the symbols that serve as labels for transition's input places, and it is an expression constructed of variables and the Boolean connectives \wedge and \vee determining the following conditions:

$\wedge(l_{i_1}, l_{i_2}, \dots, l_{i_u})$ — every place $l_{i_1}, l_{i_2}, \dots, l_{i_u}$ must contain at least one token,

$\vee(l_{i_1}, l_{i_2}, \dots, l_{i_u})$ — there must be at least one token in the set of places $l_{i_1}, l_{i_2}, \dots, l_{i_u}$, where $\{l_{i_1}, l_{i_2}, \dots, l_{i_u}\} \subset L'$.

