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# NEW MODIFICATIONS OF AN INTUITIONISTIC FUZZY IMPLICATION FROM KLEENE-DIENES TYPE. Part 2

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New intuitonistic fuzzy implications are constructed. Their relations with Modus Ponens and intuitionistic logic axioms are studied.

## 1 Introduction

The concept of *"intuitionistic fuzzy propositional calculus"* was introduced about 20 years ago (see, e.g., [1, 2]). Initially, it contained only one form of conjunction, disjunction and two forms of implication. In a series of papers a lot of new implications are defined in the frame of the intuitionistic fuzzy logic - see [1, 2, 4, 5, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20].

Here, we shall introduce three new intuitionistic fuzzy implications and will study their basic properties.

In intuitionistic fuzzy propositional calculus, if $x$ is a variable then its truth-value is represented by the ordered couple

$$V(x) = \langle a, b\rangle,$$

so that $a, b, a+b \in [0, 1]$, where $a$ and $b$ are degrees of validity and of non-validity of $x$.

Below we shall assume that for the three variables $x, y$ and $z$ the equalities: $V(x) = \langle a, b\rangle, V(y) = \langle c, d\rangle, V(z) = \langle e, f\rangle$ $(a, b, c, d, e, f, a+b, c+d, e+f \in [0, 1])$ hold.

For the needs of the discussion below we shall define the notion of Intuitionistic Fuzzy Tautology (IFT, see [1] ) by:

$$x \text{ is an IFT if and only if for } V(x) = \langle a, b\rangle \text{ holds: } a \geq b,$$

while $x$ will be a tautology iff $a = 1$ and $b = 0$. As in the case of ordinary logics, $x$ is a tautology, if $V(x) = \langle 1, 0\rangle$.

For two variables $x$ and $y$ the operations "conjunction" (&) and "disjunction" ($\vee$) are defined (see [1]) by:

$$V(x\&y) = \langle \min(a,c), \max(b,d)\rangle,$$

$$V(x \vee y) = \langle \max(a,c), \min(b,d)\rangle.$$

In [17, 18, 19] the implications $\rightarrow_{100}, \rightarrow_{101}, \ldots, \rightarrow_{105}$ are introduced by

$$V(x \rightarrow_{100} y) = \langle \max(b.\text{sg}(a), c), \min(a.\text{sg}(b), d)\rangle,$$

$$V(x \rightarrow_{101} y) = \langle \max(b.\text{sg}(a), c.\text{sg}(d)), \min(a.\text{sg}(b), d.\text{sg}(c))\rangle,$$

$$V(x \rightarrow_{102} y) = \langle \max(b, c.\text{sg}(d)), \min(a, d.\text{sg}(c))\rangle,$$

$$V(x \rightarrow_{103} y) = V(\Box x \rightarrow_{100} \Diamond y)$$

$$= \langle \max((1-a).\text{sg}(a), 1-d), \min(a.\text{sg}(1-a), d)\rangle,$$

$$V(x \rightarrow_{104} y) = V(\Box x \rightarrow_{101} \Diamond y)$$

$$= \langle \max((1-a).\text{sg}(a), (1-d).\text{sg}(d)), \min(a.\text{sg}(1-a), d.\text{sg}(1-d))\rangle,$$

$$V(x \rightarrow_{105} y) = V(\Box x \rightarrow_{102} \Diamond y)$$

$$= \langle \max(1-a, (1-d).\text{sg}(d)), \min(a, d.\text{sg}(1-d))\rangle,$$

and it is shown that their definitions are correct. There, it is discussed that these implications are modifications of Kleene-Dienes's implication (see [7, 8]) and on its basis operations "negation" can be constructed, as follows:

$$\neg\langle a,b\rangle = \langle a,b\rangle \rightarrow \langle 0,1\rangle = \langle b.\text{sg}(a), a.\text{sg}(b)\rangle.$$

In [17, 18, 19] it is proved that these negations are not classical ones.

# 2 Main results

In [21] an implication $\rightarrow_*$ is introduced on the base of another, already defined implication $\rightarrow$ by

$$x \rightarrow_* y = \Box x \rightarrow \Diamond y.$$

This idea is used in [19] for defining of implications $\rightarrow_{103}, \rightarrow_{104}, \rightarrow_{105}$.

Now, we shall use the dual form of this implication, i.e.,

$$x \rightarrow_* y = \Diamond x \rightarrow \Box y. \qquad (*)$$

Here, using (*), we shall construct three new implications and will stidy their properties that are analogous of the above ones.

**3.1** The three new implications will have the forms:

$$V(x \rightarrow_{106} y) = V(\Diamond x \rightarrow_{100} \Box y) = \langle \max(b.\text{sg}(1-b), c), \min((1-b).\text{sg}(b), 1-c)\rangle$$