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ON THE NEGATIONS OVER INTUITIONISTIC FIZZY SETS. Part 1

Krassimir Atanassov and Dimitar Dimitrov

CLBME-Bulgarian Academy of Sciences, Acad. G. Bonchev Str., Bl. 105,
Sofia-1113, Bulgaria
e-mails: *krat@bas.bg* and *mitex@gbg.bg*

Abstract

The first 99 implications and the first 17 negations over intuitionistic fuzzy sets, introduced by the moment, are collected and some of their properties are studied.

1 On some previous results

The concept of the Intuitionistic Fuzzy Set (IFS, see [1]) was introduced in 1983 as an extension of Zadeh's fuzzy set. All operations, defined over fuzzy sets were transformed for the IFS case. One of them - operartion "negation" was "the apple of discord" between spacialists interested in IFS theory. Some of them decided that the name of the concept is not suitable, because the first negation, defined over IFSs satisfies axioms of first order logic

$$X \to \neg\neg X \qquad (A1)$$

and

$$\neg\neg X \to X \qquad (A2),$$

while in intuitionistic logic Axiom A2 is not valid. On IFS we can define operation "negation" by different ways and some of the new defined operations satisfy only Axiom A1. Here, we shall collect all introduced by the moment, negations and some of their properties will be studied.

In [1] the concept of an *Intuitionistic Fuzzy Tautological Set* (IFTS) is introduced as follows: the IFS A is an IFTS iff for every $x \in E : \mu_A(x) \geq \nu_A(x)$.

In some definitions we shall use functions sg and $\overline{sg}$:

$$\mathrm{sg}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}, \qquad \overline{sg}(x) = \begin{cases} 0 & \text{if } x > 0 \\ 1 & \text{if } x \leq 0 \end{cases}$$

Let

$$E^* = \{\langle x, 1, 0\rangle | x \in E\},$$
$$O^* = \{\langle x, 0, 1\rangle | x \in E\},$$
$$U^* = \{\langle x, 0, 0\rangle | x \in E\}.$$

Let A and B be two fixed IFSs and let

$$X_i = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\} \rightarrow_i \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in E\}.$$

Now, following [2], we shall introduce 99 different IFS-implications.

$$X_1 = \langle x, \max(\nu_A(x), \min(\mu_A(x), \mu_B(x))), \min(\mu_A(x), \nu_B(x)))\rangle | x \in E\},$$

$$X_2 = \langle x, 1 - \mathrm{sg}(\mu_A(x) - \mu_B(x)), \nu_B(x).\mathrm{sg}(\mu_A(x) - \mu_B(x))\rangle | x \in E\},$$
$$X_3 = \langle x, 1 - (1 - \mu_B(x)).\mathrm{sg}(\mu_A(x) - \mu_B(x)), \nu_B(x).\mathrm{sg}(\mu_A(x) - \mu_B(x))\rangle | x \in E\},$$
$$X_4 = \langle x, \max(\nu_A(x), \mu_B(x)), \min(\mu_A(x), \nu_B(x))\rangle | x \in E\},$$
$$X_5 = \langle x, \min(1, \nu_A(x) + \mu_B(x)), \max(0, \mu_A(x) + \nu_B(x) - 1)\rangle | x \in E\},$$
$$X_6 = \langle x, \nu_A(x) + \mu_A(x).\mu_B(x), \mu_A(x).\nu_B(x)\rangle | x \in E\},$$
$$X_7 = \langle x, \min(\max(\nu_A(x), \mu_B(x)), \max(\mu_A(x), \nu_A(x)), \max(\mu_B(x), \nu_B(x))),$$
$$\max(\min(\mu_A(x), \nu_B(x)), \min(\mu_A(x), \nu_A(x)), \min(\mu_B(x), \nu_B(x)))\rangle | x \in E\},$$
$$X_8 = \langle x, 1 - (1 - \min(\nu_A(x), \mu_B(x))).\mathrm{sg}(\mu_A(x) - \mu_B(x)),$$
$$\max(\mu_A(x), \nu_B(x)).\mathrm{sg}(\mu_A(x) - \mu_B(x)).\mathrm{sg}(\nu_B(x) - \nu_A(x))\rangle | x \in E\},$$
$$X_9 = \langle x, \nu_A(x) + \mu_A(x)^2 \mu_B(x), \mu_A(x)\nu_A(x) + \mu_A(x)^2 \nu_B(x)\rangle | x \in E\},$$
$$X_{10} = \langle x, \mu_B(x).\overline{sg}(1 - \mu_A(x)) + \mathrm{sg}(1 - \mu_A(x)).(\overline{sg}(1 - \mu_B(x)) + \nu_A(x).\mathrm{sg}(1 - \mu_B(x))),$$
$$\nu_B(x).\overline{sg}(1 - \mu_A(x)) + \mu_A(x).\mathrm{sg}(1 - \mu_A(x)).\mathrm{sg}(1 - \mu_B(x))\rangle | x \in E\},$$
$$X_{11} = \langle x, 1 - (1 - \mu_B(x)).\mathrm{sg}(\mu_A(x) - \mu_B(x)),$$
$$\nu_B(x).\mathrm{sg}(\mu_A(x) - \mu_B(x)).\mathrm{sg}(\nu_B(x) - \nu_A(x))\rangle | x \in E\},$$
$$X_{12} = \langle x, \max(\nu_A(x), \mu_B(x)), 1 - \max(\nu_A(x), \mu_B(x))\rangle | x \in E\},$$
$$X_{13} = \langle x, \nu_A(x) + \mu_B(x) - \nu_A(x).\mu_B(x), \mu_A(x).\nu_B(x)\rangle | x \in E\},$$
$$X_{14} = \langle x, 1 - (1 - \mu_B(x)).\mathrm{sg}(\mu_A(x) - \mu_B(x)) - \nu_B(x).\overline{sg}(\mu_A(x) - \mu_B(x))$$
$$.\mathrm{sg}(\nu_B(x) - \nu_A(x)), \nu_B(x).\mathrm{sg}(\nu_B(x) - \nu_A(x))\rangle | x \in E\},$$
$$X_{15} = \langle x, 1 - (1 - \min(\nu_A(x), \mu_B(x))).\mathrm{sg}(\mathrm{sg}(\mu_A(x) - \mu_B(x)) + \mathrm{sg}(\nu_B(x) - \nu_A(x)))$$
$$- \min(\nu_A(x), \mu_B(x)).\mathrm{sg}(\mu_A(x) - \mu_B(x)).\mathrm{sg}(d - \nu_A(x)),$$
$$1 - (1 - \max(\mu_A(x), \nu_B(x))).\mathrm{sg}(\overline{sg}(\mu_A(x) - \mu_B(x)) + \overline{sg}(\nu_B(x) - \nu_A(x)))$$
$$- \max(\mu_A(x), \nu_B(x)).\overline{sg}(\mu_A(x) - \mu_B(x)).\overline{sg}(\nu_B(x) - \nu_A(x))\rangle | x \in E\},$$

$$X_{16} = \langle x, \max(1 - \mathrm{sg}(\mu_A(x)), \mu_B(x)), \min(\mathrm{sg}(\mu_A(x)), \nu_B(x))\rangle | x \in E\},$$

$$X_{17} = \langle x, \max(\nu_A(x), \mu_B(x)), \min(\mu_A(x).\nu_A(x) + \mu_A(x)^2, \nu_B(x))\rangle | x \in E\},$$

$$X_{18} = \langle x, \max(\nu_A(x), \mu_B(x)), \min(1 - \nu_A(x), \nu_B(x))\rangle | x \in E\},$$

$$X_{19} = \langle x, \max(1 - \mathrm{sg}(\mathrm{sg}(\mu_A(x)) + \mathrm{sg}(1 - \nu_A(x))), \mu_B(x)),$$
$$\min(\mathrm{sg}(1 - \nu_A(x)), \nu_B(x))\rangle | x \in E\},$$

$$X_{20} = \langle x, \max(1 - \mathrm{sg}(\mu_A(x)), 1 - \mathrm{sg}(1 - \mathrm{sg}(\mu_B(x)))),$$
$$\min(\mathrm{sg}(\mu_A(x)), \mathrm{sg}(1 - sg(\mu_B(x)))))\rangle | x \in E\},$$

$$X_{21} = \langle x, \max(\nu_A(x), \mu_B(x)(\mu_B(x) + \nu_B(x))),$$
$$\min(a(\mu_A(x) + \nu_A(x)), \nu_B(x)(\mu_B(x)^2 + d + \mu_B(x)\nu_B(x)))\rangle | x \in E\},$$

$$X_{22} = \langle x, \max(\nu_A(x), 1 - \nu_B(x)), \min(1 - \nu_A(x), \nu_B(x))\rangle | x \in E\},$$

$$X_{23} = \langle x, 1 - \min(\mathrm{sg}(1 - \nu_A(x)), \mathrm{sg}(1 - \mathrm{sg}(1 - \nu_B(x)))),$$
$$\min(\mathrm{sg}(1 - \nu_A(x)), \mathrm{sg}(1 - \mathrm{sg}(1 - \nu_B(x)))))\rangle | x \in E\},$$

$$X_{24} = \langle x, \overline{sg}(\mu_A(x) - \mu_B(x)).\overline{sg}(\nu_B(x) - \nu_A(x)),$$
$$\mathrm{sg}(\mu_A(x) - \mu_B(x)).\mathrm{sg}(\nu_B(x) - \nu_A(x))\rangle | x \in E\},$$

$$X_{25} = \langle x, \max(\nu_A(x).\overline{sg}(\mu_A(x)).\overline{sg}(1 - \nu_A(x)), \mu_B(x).\overline{sg}(\nu_B(x)).\overline{sg}(1 - \mu_B(x))),$$
$$\min(\mu_A(x).\mathrm{sg}(1 - \nu_A(x)), \nu_B(x).\mathrm{sg}(1 - \mu_B(x)))\rangle | x \in E\},$$

$$X_{26} = \langle x, \max(\overline{sg}(1 - \nu_A(x)), \mu_B(x)), \min(\mathrm{sg}(\mu_A(x)), \nu_B(x))\rangle | x \in E\},$$

$$X_{27} = \langle x, \max(\overline{sg}(1 - \nu_A(x)), \mathrm{sg}(\mu_B(x))), \min(\mathrm{sg}(\mu_A(x)), \overline{sg}(1 - \nu_B(x)))\rangle | x \in E\},$$

$$X_{28} = \langle x, \max(\overline{sg}(1 - \nu_A(x)), \mu_B(x)), \min(\mu_A(x), \nu_B(x))\rangle | x \in E\},$$

$$X_{29} = \langle x, \max(\overline{sg}(1 - \nu_A(x)), \overline{sg}(1 - \mu_B(x))), \min(\mu_A(x), \overline{sg}(1 - \nu_B(x)))\rangle | x \in E\},$$

$$X_{30} = \langle x, 1 - \min(\mu_A(x), \max(1 - \mu_A(x), \nu_B(x))), \min(\mu_A(x), \nu_B(x)))\rangle | x \in E\},$$

$$X_{31} = \langle x, 1 - \mathrm{sg}(\mu_A(x) + \mu_B(x) - 1), \nu_B(x).\mathrm{sg}(\mu_A(x) + \nu_B(x) - 1)\rangle | x \in E\},$$

$$X_{32} = \langle x, 1 - \nu_B(x).\mathrm{sg}(\mu_A(x) + \mu_B(x) - 1), \nu_B(x).\mathrm{sg}(\mu_A(x) + \nu_B(x) - 1)\rangle | x \in E\},$$

$$X_{33} = \langle x, 1 - \min(\mu_A(x), \nu_B(x)), \min(\mu_A(x), \nu_B(x))\rangle | x \in E\},$$

$$X_{34} = \langle x, \min(1, 2 - \mu_A(x) - \nu_B(x)), \max(0, \mu_A(x) + \nu_B(x) - 1)\rangle | x \in E\},$$

$$X_{35} = \langle x, 1 - \mu_A(x).\nu_B(x), \mu_A(x).\nu_B(x)\rangle | x \in E\},$$

$$X_{36} = \langle x, \min(1 - \min(\mu_A(x), \nu_B(x)), \max(\mu_A(x), 1 - \mu_A(x)), \max(1 - \nu_B(x), \nu_B(x))),$$
$$\max(\min(\mu_A(x), \nu_B(x)), \min(\mu_A(x), 1 - \mu_A(x)), \min(1 - \nu_B(x), \nu_B(x)))\rangle | x \in E\},$$

$$X_{37} = \langle x, 1 - \max(\mu_A(x), \nu_B(x)).\mathrm{sg}(\mu_A(x) + \nu_B(x) - 1),$$
$$\max(\mu_A(x), \nu_B(x)).\mathrm{sg}(\mu_A(x) + \nu_B(x) - 1)\rangle | x \in E\},$$

$$X_{38} = \langle x, 1 - \mu_A(x) + \mu_A(x)^2 - \mu_A(x)^2.\mu_B(x), \mu_A(x) - \mu_A(x)^2 + \mu_A(x)^2.\nu_B(x)\rangle | x \in E\},$$

$$X_{39} = \langle x, (1-\nu_B(x)).\overline{sg}(1-\mu_A(x))+\mathrm{sg}(1-\mu_A(x)).(\overline{sg}(\nu_B(x)+(1-\mu_A(x)).\mathrm{sg}(\nu_B(x)),$$
$$\nu_B(x).\overline{sg}(1-\mu_A(x)) + \mu_A(x).\mathrm{sg}(1-\mu_A(x)).\mathrm{sg}(\nu_B(x))\rangle | x \in E\},$$
$$X_{40} = \langle x, 1-\mathrm{sg}(\mu_A(x)+\nu_B(x)-1), 1-\overline{sg}(\mu_A(x)+\nu_B(x)-1)\rangle | x \in E\},$$
$$X_{41} = \langle x, 1-\min(\mathrm{sg}(\mu_A(x)), 1-\nu_B(x)), \min(\mathrm{sg}(\mu_A(x)), \nu_B(x))\rangle | x \in E\},$$
$$X_{42} = \langle x, 1-\min(\mathrm{sg}(\mu_A(x)), \overline{sg}(1-\nu_B(x))), \min(\mathrm{sg}(\mu_A(x)), \overline{sg}(1-\nu_B(x)))\rangle | x \in E\},$$
$$X_{43} = \langle x, \max(\overline{sg}(\mu_A(x)), 1-\nu_B(x)), \min(\mathrm{sg}(\mu_A(x)), \nu_B(x))\rangle | x \in E\},$$
$$X_{44} = \langle x, \max(\overline{sg}(\mu_A(x)), 1-\nu_B(x)), \min(\mu_A(x), \nu_B(x))\rangle | x \in E\},$$
$$X_{45} = \langle x, \max(\overline{sg}(\mu_A(x)), \overline{sg}(\nu_B(x))), \min(\overline{sg}(\mu_A(x)), \overline{sg}(1-\nu_B(x)))\rangle | x \in E\},$$
$$X_{46} = \langle x, \max(\nu_A(x), \min(1-\nu_A(x), \mu_B(x))), 1-\max(\nu_A(x), \mu_B(x)))\rangle | x \in E\},$$
$$X_{47} = \langle x, 1-\mathrm{sg}(1-\nu_A(x)-\mu_B(x)), (1-\mu_B(x)).\mathrm{sg}(1-\nu_A(x)-\mu_B(x))\rangle | x \in E\},$$
$$X_{48} = \langle x, 1-(1-\mu_B(x)).\mathrm{sg}(1-\nu_A(x)-\mu_B(x)),$$
$$(1-\mu_B(x)).\mathrm{sg}(1-\nu_A(x)-\mu_B(x))\rangle | x \in E\},$$
$$X_{49} = \langle x, \min(1, \nu_A(x)+\mu_B(x)), \max(0, 1-\nu_A(x)-\mu_B(x))\rangle | x \in E\},$$
$$X_{50} = \langle x, \nu_A(x)+\mu_B(x)-\nu_A(x).\mu_B(x), (1-\nu_A(x)).(1-\mu_B(x))\rangle | x \in E\},$$
$$X_{51} = \langle x, \min(\max(\nu_A(x), \mu_B(x)), \max(1-\nu_A(x), \nu_A(x)), \max(\mu_B(x), 1-\mu_B(x))),$$
$$\max(1-\max(\nu_A(x), \mu_B(x)), \min(1-\nu_A(x), \nu_A(x)), \min(\mu_B(x), 1-\mu_B(x)))\rangle | x \in E\},$$
$$X_{52} = \langle x, 1-(1-\min(\nu_A(x), \mu_B(x))).\mathrm{sg}(1-\nu_A(x)-\nu_B(x)),$$
$$(1-\min(\nu_A(x), \mu_B(x)).\mathrm{sg}(1-\nu_A(x)-\mu_B(x))\rangle | x \in E\},$$
$$X_{53} = \langle x, \nu_A(x)+(1-\nu_A(x))^2.\mu_B(x), 1-\nu_A(x)-(1-\nu_A(x))^2.\nu_B(x)\rangle | x \in E\},$$
$$X_{54} = \langle x, \mu_B(x).\overline{sg}(\nu_A(x))+\mathrm{sg}(\nu_A(x)).(\overline{sg}(1-\mu_B(x))+\nu_A(x).\mathrm{sg}(1-\mu_B(x))),$$
$$(1-\mu_B(x)).\overline{sg}(\nu_A(x))+(1-\nu_A(x)).\mathrm{sg}(\nu_A(x)).\mathrm{sg}(1-\mu_B(x))\rangle | x \in E\},$$
$$X_{55} = \langle x, 1-\mathrm{sg}(1-\nu_A(x)-\mu_B(x)), 1-\overline{sg}(1-\nu_A(x)-\mu_B(x))\rangle | x \in E\},$$
$$X_{56} = \langle x, 1-\min(\mathrm{sg}(1-\nu_B(x)), \mu_B(x)), \min(\mathrm{sg}(1-\nu_B(x)), 1-\mu_B(x))\rangle | x \in E\},$$
$$X_{57} = \langle x, 1-\min(\mathrm{sg}(1-\nu_B(x)), \overline{sg}(\mu_B(x))), \min(\mathrm{sg}(1-\nu_B(x)), \overline{sg}(\mu_B(x)))\rangle | x \in E\},$$
$$X_{58} = \langle x, \max(\overline{sg}(1-\nu_A(x)), \overline{sg}(1-\mu_B(x))), 1-\max(\nu_A(x), \mu_B(x))\rangle | x \in E\},$$
$$X_{59} = \langle x, \max(\overline{sg}(1-\nu_A(x)), \mu_B(x)), 1-\max(\nu_A(x), \mu_B(x))\rangle | x \in E\},$$
$$X_{60} = \langle x, \max(\overline{sg}(1-\nu_A(x)), \overline{sg}(1-\mu_B(x))), \min(\overline{sg}(1-\nu_A(x)), \overline{sg}(\mu_B(x)))\rangle | x \in E\},$$
$$X_{61} = \langle x, \max(\mu_B(x), \min(\nu_B(x), \nu_A(x))), \min(\nu_B(x), \mu_A(x)))\rangle | x \in E\},$$
$$X_{62} = \langle x, 1-\mathrm{sg}(\nu_B(x)-\nu_A(x)), \mu_A(x).\mathrm{sg}(\nu_B(x)-\nu_A(x))\rangle | x \in E\},$$
$$X_{63} = \langle x, 1-(1-\nu_A(x)).\mathrm{sg}(\nu_B(x)-\nu_A(x)), \mu_A(x).\mathrm{sg}(\nu_B(x)-\nu_A(x))\rangle | x \in E\},$$
$$X_{64} = \langle x, \mu_B(x)+\nu_B(x).\nu_A(x), \nu_B(x).\mu_A(x)\rangle | x \in E\},$$

$$X_{65} = \langle x, 1-(1-\min(\mu_B(x),\nu_A(x))).\mathrm{sg}(\nu_B(x)-\nu_A(x)),$$
$$\max(\nu_B(x),\mu_A(x)).\mathrm{sg}(\nu_B(x)-\nu_A(x)).\mathrm{sg}(\mu_A(x)-\mu_B(x))\rangle | x\in E\},$$
$$X_{66} = \langle x, \mu_B(x)+\nu_B(x)^2\nu_A(x), \nu_B(x)\mu_B(x)+\nu_B(x)^2\mu_A(x)\rangle | x\in E\},$$
$$X_{67} = \langle x, \nu_A(x).\overline{sg}(1-\nu_B(x))+\mathrm{sg}(1-\nu_B(x)).(\overline{sg}(1-\nu_A(x))+\mu_B(x).\mathrm{sg}(1-\nu_A(x))),$$
$$\mu_A(x).\overline{sg}(1-\nu_B(x))+\nu_B(x).\mathrm{sg}(1-\nu_B(x)).\mathrm{sg}(1-\nu_A(x))\rangle | x\in E\},$$
$$X_{68} = \langle x, 1-(1-\nu_A(x)).\mathrm{sg}(\nu_B(x)-\nu_A(x)),$$
$$\mu_A(x).\mathrm{sg}(\nu_B(x)-\nu_A(x)).\mathrm{sg}(\mu_A(x)-\mu_B(x))\rangle | x\in E\},$$
$$X_{69} = \langle x, 1-(1-\nu_A(x)).\mathrm{sg}(\nu_B(x)-\nu_A(x))-\mu_A(x).\overline{sg}(\nu_B(x)-\nu_A(x))$$
$$.\mathrm{sg}(\mu_A(x)-\mu_B(x)), \mu_A(x).\mathrm{sg}(\mu_A(x)-\mu_B(x))\rangle | x\in E\},$$
$$X_{70} = \langle x, \max(1-\mathrm{sg}(\nu_B(x)),\nu_A(x)), \min(\mathrm{sg}(\nu_B(x)),\mu_A(x))\rangle | x\in E\},$$
$$X_{71} = \langle x, \max(\mu_B(x),\nu_A(x)), \min(\nu_B(x).\mu_B(x)+\nu_B(x)^2,\mu_A(x))\rangle | x\in E\},$$
$$X_{72} = \langle x, \max(\mu_B(x),\nu_A(x)), \min(1-\mu_B(x),\mu_A(x))\rangle | x\in E\},$$
$$X_{73} = \langle x, \max(1-\mathrm{sg}(\mathrm{sg}(\nu_B(x))+\mathrm{sg}(1-\mu_B(x))),\nu_A(x)),$$
$$\min(\mathrm{sg}(1-\mu_B(x)),\mu_A(x))\rangle | x\in E\},$$
$$X_{74} = \langle x, \max(1-\mathrm{sg}(\nu_B(x)),1-\mathrm{sg}(1-\mathrm{sg}(\nu_A(x)))),$$
$$\min(\mathrm{sg}(\nu_B(x)),\mathrm{sg}(1-sg(\nu_A(x))))\rangle | x\in E\},$$
$$X_{75} = \langle x, \max(\mu_B(x),\nu_A(x)(\nu_A(x)+\mu_A(x))),$$
$$\min(a(\nu_B(x)+\mu_B(x)),\mu_A(x)(\nu_A(x)^2+d+\nu_A(x)\mu_A(x)))\rangle | x\in E\},$$
$$X_{76} = \langle x, \max(\mu_B(x),1-\mu_A(x)), \min(1-\mu_B(x),\mu_A(x))\rangle | x\in E\},$$
$$X_{77} = \langle x, 1-\min(\mathrm{sg}(1-\mu_B(x)),\mathrm{sg}(1-\mathrm{sg}(1-\mu_A(x)))),$$
$$\min(\mathrm{sg}(1-\mu_B(x)),\mathrm{sg}(1-\mathrm{sg}(1-\mu_A(x))))\rangle | x\in E\},$$
$$X_{78} = \langle x, \max(\overline{sg}(1-\mu_B(x)),\nu_A(x)), \min(\mu_A(x),\mathrm{sg}(\nu_B(x)))\rangle | x\in E\},$$
$$X_{79} = \langle x, \max(\overline{sg}(1-\mu_B(x)),\mathrm{sg}(\nu_A(x))), \min(\mathrm{sg}(\nu_B(x)),\overline{sg}(1-\mu_A(x)))\rangle | x\in E\},$$
$$X_{80} = \langle x, \max(\overline{sg}(1-\mu_B(x)),\nu_A(x)), \min(\mu_A(x),\nu_B(x))\rangle | x\in E\},$$
$$X_{81} = \langle x, \max(\overline{sg}(1-\mu_B(x)),\overline{sg}(1-\nu_A(x))),$$
$$\min(\overline{sg}(1-\mu_A(x)),\overline{sg}(1-\mu_B(x)))\rangle | x\in E\},$$
$$X_{82} = \langle x, 1-\min(\nu_B(x),\max(1-\nu_B(x),\mu_A(x))),$$
$$\min(\nu_B(x),\mu_A(x)))\rangle | x\in E\},$$
$$X_{83} = \langle x, 1-\mathrm{sg}(\nu_B(x)+\nu_A(x)-1), \mu_A(x).\mathrm{sg}(\nu_B(x)+\mu_A(x)-1)\rangle | x\in E\},$$
$$X_{84} = \langle x, 1-\mu_A(x).\mathrm{sg}(\nu_B(x)+\nu_A(x)-1), \mu_A(x).\mathrm{sg}(\nu_B(x)+\mu_A(x)-1)\rangle | x\in E\},$$
$$X_{85} = \langle x, 1-\nu_B(x)+\nu_B(x)^2-\nu_B(x)^2.\nu_A(x), \nu_B(x)-\nu_B(x)^2+\nu_B(x)^2.\mu_A(x)\rangle | x\in E\},$$

$$X_{86} = \langle x, (1-\mu_A(x)).\overline{sg}(1-\nu_B(x))+\mathrm{sg}(1-\nu_B(x)).(\overline{sg}(\mu_A(x)+(1-\nu_B(x)).\mathrm{sg}(\mu_A(x)),$$
$$\mu_A(x).\overline{sg}(1-\nu_B(x)) + \nu_B(x).\mathrm{sg}(1-\nu_B(x)).\mathrm{sg}(\mu_A(x))\rangle | x \in E\},$$
$$X_{87} = \langle x, 1 - \min(\mathrm{sg}(\nu_B(x)), 1 - \mu_A(x)), \min(\mathrm{sg}(\nu_B(x)), \mu_A(x))\rangle | x \in E\},$$
$$X_{88} = \langle x, 1 - \min(\mathrm{sg}(\nu_B(x)), \overline{sg}(1 - \mu_A(x))), \min(\mathrm{sg}(\nu_B(x)), \overline{sg}(1 - \mu_A(x)))\rangle | x \in E\},$$
$$X_{89} = \langle x, \max(\overline{sg}(\nu_B(x)), 1 - \mu_A(x)), \min(\mu_A(x)), \nu_B(x))\rangle | x \in E\},$$
$$X_{90} = \langle x, \max(\overline{sg}(\mu_A(x)), \overline{sg}(\nu_B(x))), \min(\overline{sg}(\nu_B(x)), \overline{sg}(1 - \mu_A(x)))\rangle | x \in E\},$$
$$X_{91} = \langle x, \max(\mu_B(x), \min(1 - \mu_B(x), \nu_A(x))), 1 - \max(\mu_B(x), \nu_A(x)))\rangle | x \in E\},$$
$$X_{92} = \langle x, 1 - \mathrm{sg}(1 - \mu_B(x) - \nu_A(x)), (1 - \nu_A(x)).\mathrm{sg}(1 - \mu_B(x) - \nu_A(x))\rangle | x \in E\},$$
$$X_{93} = \langle x, 1 - (1 - \nu_A(x)).\mathrm{sg}(1 - \mu_B(x) - \nu_A(x)),$$
$$(1 - \nu_A(x)).\mathrm{sg}(1 - \mu_B(x) - \nu_A(x))\rangle | x \in E\},$$
$$X_{94} = \langle x, \mu_B(x) + (1 - \mu_B(x))^2.\nu_A(x),$$
$$1 - \mu_B(x) - (1 - \mu_B(x))^2.\mu_A(x)\rangle | x \in E\},$$
$$X_{95} = \langle x, \nu_A(x).\overline{sg}(\mu_B(x)) + \mathrm{sg}(\mu_B(x)).(\overline{sg}(1 - \nu_A(x)) + \mu_B(x).\mathrm{sg}(1 - \nu_A(x))),$$
$$(1 - \nu_A(x)).\overline{sg}(\mu_B(x)) + (1 - \mu_B(x)).\mathrm{sg}(\mu_B(x)).\mathrm{sg}(1 - \nu_A(x))\rangle | x \in E\},$$
$$X_{96} = \langle x, 1 - \min(\mathrm{sg}(1 - \mu_A(x)), \nu_A(x)), \min(\mathrm{sg}(1 - \mu_A(x)), 1 - \nu_A(x))\rangle | x \in E\},$$
$$X_{97} = \langle x, 1 - \min(\mathrm{sg}(1 - \mu_A(x)), \overline{sg}(\nu_A(x))), \min(\mathrm{sg}(1 - \mu_A(x)), \overline{sg}(\nu_A(x)))\rangle | x \in E\},$$
$$X_{98} = \langle x, \max(\overline{sg}(1 - \mu_B(x)), \nu_A(x)), 1 - \max(\mu_B(x), \nu_A(x))\rangle | x \in E\},$$
$$X_{99} = \langle x, \max(\overline{sg}(1 - \mu_B(x)), \overline{sg}(1 - \nu_A(x))), \min(\overline{sg}(1 - \mu_B(x)), \overline{sg}(\nu_A(x)))\rangle | x \in E\}.$$

2 Main results

Now, we shall use the formula (see, e.g., [3]):

$$\neg X = X \to 0$$

that in the IFS case (when an IFS A is given) has the form

$$\neg A = A \to O^*$$

and from the above 99 implications we shall obtain 18 different negations, as follows:

$$\neg_1 A = \{\langle \nu_A(x), \mu_A(x)\rangle | x \in E\},$$
$$\neg_2 A = \{\langle \overline{sg}(\mu_A(x)), \mathrm{sg}(\mu_A(x))\rangle | x \in E\},$$
$$\neg_3 A = \{\langle \nu_A(x), \mu_A(x).\nu_A(x) + \mu_A(x)^2\rangle | x \in E\},$$
$$\neg_4 A = \{\langle \nu_A(x), 1 - \nu_A(x)\rangle | x \in E\},$$
$$\neg_5 A = \{\langle \overline{sg}(1 - \nu_A(x)), \mathrm{sg}(1 - \nu_A(x))\rangle | x \in E\},$$
$$\neg_6 A = \{\langle \overline{sg}(1 - \nu_A(x)), \mathrm{sg}(\mu_A(x))\rangle | x \in E\},$$
$$\neg_7 A = \{\langle \overline{sg}(1 - \nu_A(x)), \mu_A(x)\rangle | x \in E\},$$

$$\neg_8 A = \{\langle 1-\mu_A(x), \mu_A(x)\rangle | x \in E\},$$
$$\neg_9 A = \{\langle \overline{sg}(\mu_A(x)), \mu_A(x)\rangle | x \in E\},$$
$$\neg_{10} A = \{\langle \overline{sg}(1-\nu_A(x)), 1-\nu_A(x)\rangle | x \in E\},$$
$$\neg_{11} A = \{\langle \mathrm{sg}(\nu_A(x)), \overline{sg}(\nu_A(x))\rangle | x \in E\},$$
$$\neg_{12} A = \{\langle \nu_A(x)(\mu_A(x)+\nu_A(x)), \mu_A(x)(a+\nu_A(x)+\nu_A(x)^2)\rangle | x \in E\},$$
$$\neg_{13} A = \{\langle \mathrm{sg}(1-\nu_A(x)), \overline{sg}(1-\mu_A(x))\rangle | x \in E\},$$
$$\neg_{14} A = \{\langle \mathrm{sg}(\nu_A(x)), \overline{sg}(1-\mu_A(x))\rangle | x \in E\},$$
$$\neg_{15} A = \{\langle \overline{sg}(1-\nu_A(x)), \overline{sg}(1-\mu_A(x))\rangle | x \in E\},$$
$$\neg_{16} A = \{\langle \overline{sg}(\mu_A(x)), \overline{sg}(1-\mu_A(x))\rangle | x \in E\},$$
$$\neg_{17} A = \{\langle \overline{sg}(1-\nu_A(x)), \overline{sg}(\nu_A(x))\rangle | x \in E\}.$$

We must note that some implications generate equal negations. In Table 1 is given the list of implications that generate i-th negation.

Table 1

Negation	Implications which generate the negation
$\neg_1$	$\rightarrow_1, \rightarrow_4, \rightarrow_5, \rightarrow_6, \rightarrow_7, \rightarrow_{10}, \rightarrow_{13}, \rightarrow_{61}, \rightarrow_{63}, \rightarrow_{64}, \rightarrow_{66}, \rightarrow_{67}, \rightarrow_{68}, \rightarrow_{69}, \rightarrow_{70}, \rightarrow_{71}, \rightarrow_{72}, \rightarrow_{73}, \rightarrow_{78}, \rightarrow_{80}$
$\neg_2$	$\rightarrow_2, \rightarrow_3, \rightarrow_8, \rightarrow_{11}, \rightarrow_{16}, \rightarrow_{20}, \rightarrow_{31}, \rightarrow_{32}, \rightarrow_{37}, \rightarrow_{40}, \rightarrow_{41}, \rightarrow_{42}$
$\neg_3$	$\rightarrow_9, \rightarrow_{17}, \rightarrow_{21}$
$\neg_4$	$\rightarrow_{12}, \rightarrow_{18}, \rightarrow_{22}, \rightarrow_{46}, \rightarrow_{49}, \rightarrow_{50}, \rightarrow_{51}, \rightarrow_{53}, \rightarrow_{54}, \rightarrow_{91}, \rightarrow_{93}, \rightarrow_{94}, \rightarrow_{95}, \rightarrow_{96}, \rightarrow_{98}$
$\neg_5$	$\rightarrow_{14}, \rightarrow_{15}, \rightarrow_{19}, \rightarrow_{23}, \rightarrow_{47}, \rightarrow_{48}, \rightarrow_{52}, \rightarrow_{55}, \rightarrow_{56}, \rightarrow_{57}$
$\neg_6$	$\rightarrow_{24}, \rightarrow_{26}, \rightarrow_{27}, \rightarrow_{65}$
$\neg_7$	$\rightarrow_{25}, \rightarrow_{28}, \rightarrow_{29}, \rightarrow_{62}$
$\neg_8$	$\rightarrow_{30}, \rightarrow_{33}, \rightarrow_{34}, \rightarrow_{35}, \rightarrow_{36}, \rightarrow_{38}, \rightarrow_{39}, \rightarrow_{76}, \rightarrow_{82}, \rightarrow_{84}, \rightarrow_{85}, \rightarrow_{86}, \rightarrow_{87}, \rightarrow_{89}$
$\neg_9$	$\rightarrow_{43}, \rightarrow_{44}, \rightarrow_{45}, \rightarrow_{83}$
$\neg_{10}$	$\rightarrow_{58}, \rightarrow_{59}, \rightarrow_{60}, \rightarrow_{92}$
$\neg_{11}$	$\rightarrow_{74}, \rightarrow_{97}$
$\neg_{12}$	$\rightarrow_{75}$
$\neg_{13}$	$\rightarrow_{77}, \rightarrow_{88}$
$\neg_{14}$	$\rightarrow_{79}$
$\neg_{15}$	$\rightarrow_{81}$
$\neg_{16}$	$\rightarrow_{90}$
$\neg_{17}$	$\rightarrow_{99}$

Now, we shall study the following properties of an IFS A:
Property $P1_{IFTS}$: $A \to \neg\neg A$ is an IFTS,
Property $P1_{standard}$: $A \to \neg\neg A = E^*$,
Property $P2_{IFTS}$: $\neg\neg A \to A$ is an IFTS,
Property $P2_{standard}$: $\neg\neg A \to A = E^*$,
Property $P3$: $\neg\neg\neg A = \neg A$.

In Table 2 we give all couples $(\neg, \to)$ and the list of above properties that they satisfy (marked there by "+".

Table 2

Negation	Implication	$P1_{IFTS}$	$P1_{standard}$	$P2_{IFTS}$	$P2_{standard}$	$P3$
$\neg_1$	$\to_1$	+	+			+
$\neg_1$	$\to_4$	+	+			+
$\neg_1$	$\to_5$	+	+			+
$\neg_1$	$\to_6$	+	+			+
$\neg_1$	$\to_7$	+	+			+
$\neg_1$	$\to_{10}$					+
$\neg_1$	$\to_{13}$	+	+			+
$\neg_1$	$\to_{61}$	+	+			+
$\neg_1$	$\to_{63}$	+	+	+	+	+
$\neg_1$	$\to_{64}$	+	+			+
$\neg_1$	$\to_{66}$	+	+			+
$\neg_1$	$\to_{67}$					+
$\neg_1$	$\to_{68}$	+	+	+	+	+
$\neg_1$	$\to_{69}$	+	+	+	+	+
$\neg_1$	$\to_{70}$					+
$\neg_1$	$\to_{71}$	+	+			+
$\neg_1$	$\to_{72}$	+	+			+
$\neg_1$	$\to_{73}$					+
$\neg_1$	$\to_{78}$					+
$\neg_1$	$\to_{80}$	+	+			+
$\neg_2$	$\to_2$	+		+		+
$\neg_2$	$\to_3$	+		+		+
$\neg_2$	$\to_8$	+		+		+
$\neg_2$	$\to_{11}$	+		+		+
$\neg_2$	$\to_{16}$	+		+		+
$\neg_2$	$\to_{20}$	+	+	+	+	+
$\neg_2$	$\to_{31}$	+		+		+
$\neg_2$	$\to_{32}$	+		+		+
$\neg_2$	$\to_{37}$	+		+		+
$\neg_2$	$\to_{40}$	+		+		+
$\neg_2$	$\to_{41}$	+		+		+
$\neg_2$	$\to_{42}$	+	+	+	+	+

Negation	Implication	$P1_{IFTS}$	$P1_{standard}$	$P2_{IFTS}$	$P2_{standard}$	$P3$
$\neg_3$	$\to_9$	+	+			
$\neg_3$	$\to_{17}$	+	+			
$\neg_3$	$\to_{21}$	+	+			
$\neg_4$	$\to_{12}$	+				+
$\neg_4$	$\to_{18}$	+	+			+
$\neg_4$	$\to_{22}$	+	+			+
$\neg_4$	$\to_{46}$	+				+
$\neg_4$	$\to_{49}$	+		+		+
$\neg_4$	$\to_{50}$	+				+
$\neg_4$	$\to_{51}$	+				+
$\neg_4$	$\to_{53}$	+				+
$\neg_4$	$\to_{54}$					+
$\neg_4$	$\to_{91}$	+				+
$\neg_4$	$\to_{93}$	+		+		+
$\neg_4$	$\to_{94}$	+				+
$\neg_4$	$\to_{95}$					+
$\neg_4$	$\to_{96}$					+
$\neg_4$	$\to_{98}$	+				+
$\neg_5$	$\to_{14}$	+		+		+
$\neg_5$	$\to_{15}$	+		+		+
$\neg_5$	$\to_{19}$	+		+		+
$\neg_5$	$\to_{23}$	+	+	+	+	+
$\neg_5$	$\to_{47}$	+		+		+
$\neg_5$	$\to_{48}$	+		+		+
$\neg_5$	$\to_{52}$	+		+		+
$\neg_5$	$\to_{55}$	+		+		+
$\neg_5$	$\to_{56}$	+		+		+
$\neg_5$	$\to_{57}$	+		+		+
$\neg_6$	$\to_{24}$	+		+		+
$\neg_6$	$\to_{26}$	+				+
$\neg_6$	$\to_{27}$	+	+			+
$\neg_6$	$\to_{65}$	+		+		+
$\neg_7$	$\to_{25}$	+	+			
$\neg_7$	$\to_{28}$	+	+			
$\neg_7$	$\to_{29}$	+	+			
$\neg_7$	$\to_{62}$	+	+	+		
$\neg_8$	$\to_{30}$	+	+			+
$\neg_8$	$\to_{33}$	+	+			+
$\neg_8$	$\to_{34}$	+	+	+	+	+
$\neg_8$	$\to_{35}$	+	+			+
$\neg_8$	$\to_{36}$	+	+			+

Negation	Implication	$P1_{IFTS}$	$P1_{standard}$	$P2_{IFTS}$	$P2_{standard}$	$P3$
$\neg_8$	$\rightarrow_{38}$	+	+			+
$\neg_8$	$\rightarrow_{39}$					+
$\neg_8$	$\rightarrow_{76}$	+	+			+
$\neg_8$	$\rightarrow_{82}$	+	+			+
$\neg_8$	$\rightarrow_{84}$	+	+	+	+	+
$\neg_8$	$\rightarrow_{85}$	+	+			+
$\neg_8$	$\rightarrow_{86}$					+
$\neg_8$	$\rightarrow_{87}$					+
$\neg_8$	$\rightarrow_{89}$	+	+			+
$\neg_9$	$\rightarrow_{43}$	+		+		
$\neg_9$	$\rightarrow_{44}$	+		+		
$\neg_9$	$\rightarrow_{45}$	+	+	+		
$\neg_9$	$\rightarrow_{83}$	+		+		
$\neg_{10}$	$\rightarrow_{58}$					+
$\neg_{10}$	$\rightarrow_{59}$					+
$\neg_{10}$	$\rightarrow_{60}$					+
$\neg_{10}$	$\rightarrow_{92}$					+
$\neg_{11}$	$\rightarrow_{74}$	+	+	+	+	+
$\neg_{11}$	$\rightarrow_{97}$	+		+		+
$\neg_{12}$	$\rightarrow_{75}$	+	+			
$\neg_{13}$	$\rightarrow_{77}$	+	+	+	+	+
$\neg_{13}$	$\rightarrow_{88}$	+	+	+	+	+
$\neg_{14}$	$\rightarrow_{79}$	+	+			+
$\neg_{15}$	$\rightarrow_{81}$	+	+			+
$\neg_{16}$	$\rightarrow_{99}$	+				
$\neg_{17}$	$\rightarrow_{90}$	+		+		

3 Conclusion

In the second part of the research we shall study the properties of new implications and negations defined over the IFSs.

References

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[3] Mendelson, E., *Introduction to mathematical logic,* Princeton, New Jersey, 1964.