

INTUITIONISTIC FUZZY CONJUNCTIONS AND DISJUNCTIONS FROM FIRST TYPE

Nora Angelova and Miroslav Stoenchev

Department of Bioinformatics and Mathematical Modelling
 Institute of Biophysics and Biomedical Engineering
 Bulgarian Academy of Sciences, Sofia, Bulgaria
 E-mail: miroslav@biomed.bas.bg

Abstract: The purpose of this article is the calculation of the intuitionistic fuzzy conjunctions and disjunctions with respect to certain IF implications and negations. With the constructed operations algebraic structures such as monoids, lattices, etc. may be considered and their properties studied.

Keywords: Intuitionistic fuzzy conjunctions and disjunctions.

1 Introduction

In a series of papers, 185 intuitionistic fuzzy implications were defined [1, 2, 3]. They were collected in [5]. All notations used in the present paper are from [4]. Our paper is the first part of our research. In it we described the intuitionistic fuzzy conjunctions and disjunctions that are obtained by formulas:

$$\langle a, b \rangle \vee \langle c, d \rangle = \neg \langle a, b \rangle \rightarrow \langle c, d \rangle$$

$$\langle a, b \rangle \wedge \langle c, d \rangle = \neg(\langle a, b \rangle \rightarrow \neg \langle c, d \rangle)$$

2 Construction of conjunctions and disjunctions

$\vee_{1,1}$	$\langle \min(a, c), \max(b, \min(a, d)) \rangle$
$\wedge_{1,1}$	$\langle \max(a, \min(b, c)), \min(b, d) \rangle$
$\vee_{1,2}$	$\langle \overline{\text{sg}}(\overline{\text{sg}}(a) - c), d.\text{sg}(\overline{\text{sg}}(a) - c) \rangle$
$\wedge_{1,2}$	$\langle \overline{\text{sg}}(a - \overline{\text{sg}}(c)), \text{sg}(a - \overline{\text{sg}}(c)) \rangle$
$\vee_{1,3}$	$\langle 1 - (1 - c).\text{sg}(\overline{\text{sg}}(a) - c), d.\text{sg}(\overline{\text{sg}}(a) - c) \rangle$
$\wedge_{1,3}$	$\langle \text{sg}(\text{sg}(c)\text{sg}(a - \overline{\text{sg}}(c))), \overline{\text{sg}}(\text{sg}(c)\text{sg}(a - \overline{\text{sg}}(c))) \rangle$

$\vee_{1,4}$	$\langle \max(a, c), \min(b, d) \rangle$
$\wedge_{1,4}$	$\langle \min(a, c), \max(b, d) \rangle$
$\vee_{1,5}$	$\langle \min(1, a + c), \max(0, b + d - 1) \rangle$
$\wedge_{1,5}$	$\langle \max(0, a + c - 1), \min(1, d + b) \rangle$
$\vee_{1,6}$	$\langle a + b.c, b.d \rangle$
$\wedge_{1,6}$	$\langle ac, b + ad \rangle$
$\vee_{1,7}$	$\langle \min(\max(a, b), \max(a, c), \max(c, d)), \max(\min(c, d), \min(d, b), \min(a, b)) \rangle$
$\wedge_{1,7}$	$\langle \max(\min(c, d), \min(a, c), \min(a, b)), \min(\max(a, b), \max(b, d), \max(c, d)) \rangle$
$\vee_{1,8}$	$\langle 1 - \min(1 - \min(\text{sg}(a), \text{sg}(c)), \text{sg}(\overline{\text{sg}}(a) - \text{sg}(c))),$ $\min(\max(\overline{\text{sg}}(a), \overline{\text{sg}}(c)), \text{sg}(\overline{\text{sg}}(a) - \text{sg}(c)), \text{sg}(\overline{\text{sg}}(c) - \text{sg}(a))) \rangle$
$\wedge_{1,8}$	$\langle \text{sg}((1 - \min(b, \overline{\text{sg}}(c)))\text{sg}(a - \overline{\text{sg}}(c))), \overline{\text{sg}}((1 - \min(b, \overline{\text{sg}}(c)))\text{sg}(a - \overline{\text{sg}}(c))) \rangle$
$\vee_{1,9}$	$\langle ab + a^2 + b^2c, ab^2 + ba^2 + b^2d \rangle$
$\wedge_{1,9}$	$\langle ab + a^2(cd + c^2), (b + a^2d)(ab + a^2(cd + c^2)) + (b + a^2d)^2 \rangle$
$\vee_{1,10}$	$\langle \min(c, \overline{\text{sg}}(1 - b)) + \text{sg}(1 - b)(\overline{\text{sg}}(1 - c) + \min(a, \text{sg}(1 - c))),$ $\min(d, \overline{\text{sg}}(1 - b)) + \min(b, \text{sg}(1 - b), \text{sg}(1 - c)) \rangle$
$\wedge_{1,10}$	$\langle c.\overline{\text{sg}}(1 - a) + a.\text{sg}(1 - a).\text{sg}(1 - d),$ $d.\overline{\text{sg}}(1 - a) + \text{sg}(1 - a).\overline{\text{sg}}(1 - d) + b.\text{sg}(1 - d) \rangle$
$\vee_{1,11}$	$\langle (1 - (1 - c).\text{sg}(\overline{\text{sg}}(a) - c), d.\text{sg}(\overline{\text{sg}}(a) - c).\text{sg}(d - \text{sg}(a))) \rangle$
$\wedge_{1,11}$	$\langle \text{sg}(\text{sg}(c)\text{sg}(a - \overline{\text{sg}}(c))), \overline{\text{sg}}(\text{sg}(c)\text{sg}(a - \overline{\text{sg}}(c))) \rangle$
$\vee_{1,12}$	$\langle \max(1 - b, c), 1 - \max(1 - b, c) \rangle$
$\wedge_{1,12}$	$\langle 1 - \max(b, d), \max(b, d) \rangle$
$\vee_{1,13}$	$\langle a + c - ac, bd \rangle$
$\wedge_{1,13}$	$\langle ac, b + d - bd \rangle$
$\vee_{1,14}$	$\langle c.\text{sg}(1 - c)\overline{\text{sg}}(1 - b) + \text{sg}(1 - b), d.\text{sg}(d).\text{sg}(1 - c).\overline{\text{sg}}(1 - b) \rangle$
$\wedge_{1,14}$	$\langle \overline{\text{sg}}(1 - \text{sg}(1 - d))\text{sg}(\text{sg}(1 - d) - b), \text{sg}(1 - \text{sg}(1 - d))\text{sg}(\text{sg}(1 - d) - b) \rangle$
$\vee_{1,15}$	$\langle \overline{\text{sg}}(d - b) + c.\text{sg}(d - b), d.\text{sg}(d - b) \rangle$
$\wedge_{1,15}$	$\langle \text{sg}((1 - \max(a, \text{sg}(1 - d)))\text{sg}(\overline{\text{sg}}(a - \overline{\text{sg}}(1 - d))))),$ $\overline{\text{sg}}((1 - \max(a, \text{sg}(1 - d)))\text{sg}(\overline{\text{sg}}(a - \overline{\text{sg}}(1 - d)))) \rangle$
$\vee_{1,16}$	$\langle \max(\text{sg}(a), c), \min(\overline{\text{sg}}(a), d) \rangle$
$\wedge_{1,16}$	$\langle \overline{\text{sg}}(\max(\overline{\text{sg}}(a), \overline{\text{sg}}(c))), \text{sg}(\max(\overline{\text{sg}}(a), \overline{\text{sg}}(c))) \rangle$
$\vee_{1,17}$	$\langle \max(a^2 + ab, c), \min((ab + a^2)b + b^2, d) \rangle$
$\wedge_{1,17}$	$\langle \min(ab + a^2, cd + c^2), \max(b, d) \min(ab + a^2, cd + c^2) + \max(b, d)^2 \rangle$
$\vee_{1,18}$	$\langle \max(1 - b, c), \min(b, d) \rangle$
$\wedge_{1,18}$	$\langle 1 - \max(b, d), \max(b, d) \rangle$
$\vee_{1,19}$	$\langle \max(\text{sg}(1 - b), c), \min(\overline{\text{sg}}(1 - b), d) \rangle$
$\wedge_{1,19}$	$\langle \overline{\text{sg}}(1 - \min(\text{sg}(1 - b), \text{sg}(1 - d))), \text{sg}(1 - \min(\text{sg}(1 - b), \text{sg}(1 - d))) \rangle$
$\vee_{1,20}$	$\langle \max(\text{sg}(a), \text{sg}(c)), \min(\overline{\text{sg}}(a), \overline{\text{sg}}(c)) \rangle$
$\wedge_{1,20}$	$\langle \overline{\text{sg}}(\max(\overline{\text{sg}}(a), \overline{\text{sg}}(c))), \text{sg}(\max(\overline{\text{sg}}(a), \overline{\text{sg}}(c))) \rangle$
$\vee_{1,21}$	$\langle \max(a^2 + ab, c^2 + cd), \min(b(a^2 + ab + b), d(c^2 + cd + d)) \rangle$
$\wedge_{1,21}$	$\langle \min((a(a + b)), c(c + d)(d^2 + c(c + d) + cd(c + d))),$ $\max(b, d(d + c(c + d)))(\max(b, d(d + c(c + d)))) \rangle$

	$\langle + \min(a(a+b), c(c+d)(d^2 + c(c+d) + cd(c+d))) \rangle$
$\vee_{1,22}$	$\langle 1 - \min(b, d), \min(b, d) \rangle$
$\wedge_{1,22}$	$\langle 1 - \max(b, d), \max(b, d) \rangle$
$\vee_{1,23}$	$\langle 1 - \min(\text{sg}(1-a), \overline{\text{sg}}(1-d)), \min(\text{sg}(1-a), \overline{\text{sg}}(1-d)) \rangle$
$\wedge_{1,23}$	$\langle \overline{\text{sg}}(1 - \min(\text{sg}(1-b), \text{sg}(1-d))), \text{sg}(1 - \min(\text{sg}(1-b), \text{sg}(1-d))) \rangle$
$\vee_{1,24}$	$\langle \overline{\text{sg}}(\overline{\text{sg}}(1-b) - c)\overline{\text{sg}}(d - \text{sg}(a)), \text{sg}(\overline{\text{sg}}(1-b) - c)\text{sg}(d - \text{sg}(a)) \rangle$
$\wedge_{1,24}$	$\langle \overline{\text{sg}}(1 - \text{sg}(a - \overline{\text{sg}}(1-d))\text{sg}(\text{sg}(c) - b)), \text{sg}(\overline{\text{sg}}(a - \overline{\text{sg}}(1-d))\overline{\text{sg}}(\text{sg}(c) - b)) \rangle$
$\vee_{1,25}$	$\langle \max(a, \text{sg}(1-b)\overline{\text{sg}}(1-a)), c\overline{\text{sg}}(d)\overline{\text{sg}}(1-c), \min(\overline{\text{sg}}(1-b), d) \rangle$
$\wedge_{1,25}$	$\langle \overline{\text{sg}}(1 - \min(a, c)), \max(b, (1-a)b) \rangle$
$\vee_{1,26}$	$\langle \max(\text{sg}(a), \text{sg}(c)), \min(\overline{\text{sg}}(1-b), \overline{\text{sg}}(1-d)) \rangle$
$\wedge_{1,26}$	$\langle \overline{\text{sg}}(1 - \min(\text{sg}(a), \text{sg}(c))), \text{sg}(\max(\overline{\text{sg}}(1-b), \overline{\text{sg}}(1-d))) \rangle$
$\vee_{1,27}$	$\langle \max(\text{sg}(a), \text{sg}(c)), \min(\overline{\text{sg}}(1-b), \overline{\text{sg}}(1-d)) \rangle$
$\wedge_{1,27}$	$\langle \min(\text{sg}(a), \text{sg}(c)), \max(\overline{\text{sg}}(1-b), \overline{\text{sg}}(1-d)) \rangle$
$\vee_{1,28}$	$\langle \max(\overline{\text{sg}}(1-a), \overline{\text{sg}}(1-c)), \min(\overline{\text{sg}}(1-b), \overline{\text{sg}}(1-d)) \rangle$
$\wedge_{1,28}$	$\langle \overline{\text{sg}}(1 - \min(a, c)), \max(b, \overline{\text{sg}}(1-d)) \rangle$
$\vee_{1,29}$	$\langle \max(\overline{\text{sg}}(1-a), \overline{\text{sg}}(1-c)), \min(\overline{\text{sg}}(1-b), \overline{\text{sg}}(1-d)) \rangle$
$\wedge_{1,29}$	$\langle \overline{\text{sg}}(1 - \min(a, \overline{\text{sg}}(1-c))), \max(b, \overline{\text{sg}}(1-d)) \rangle$
$\vee_{1,30}$	$\langle \max(a, 1 - \max(a, d)), \min(1 - a, d) \rangle$
$\wedge_{1,30}$	$\langle 1 - \max(1 - a, \min(a, 1 - c)), \max(1 - a, \min(a, 1 - c)) \rangle$
$\vee_{1,31}$	$\langle \overline{\text{sg}}(\overline{\text{sg}}(a) - \text{sg}(c)), \min(\overline{\text{sg}}(c), \text{sg}(\overline{\text{sg}}(a) - \text{sg}(c))) \rangle$
$\wedge_{1,31}$	$\langle \text{sg}(a + \text{sg}(c) - 1), \overline{\text{sg}}(a + \text{sg}(c) - 1) \rangle$
$\vee_{1,32}$	$\langle 1 - \min(\overline{\text{sg}}(c), \text{sg}(\overline{\text{sg}}(a) - \text{sg}(c))), \min(\overline{\text{sg}}(c), \text{sg}(\overline{\text{sg}}(a) - \text{sg}(c))) \rangle$
$\wedge_{1,32}$	$\langle \text{sg}(\text{sg}(c)\text{sg}(a + \text{sg}(c) - 1)), \overline{\text{sg}}(\text{sg}(c)\text{sg}(a + \text{sg}(c) - 1)) \rangle$
$\vee_{1,33}$	$\langle 1 - \min(1 - a, d), \min(1 - a, d) \rangle$
$\wedge_{1,33}$	$\langle \min(a, c), 1 - \min(a, c) \rangle$
$\vee_{1,34}$	$\langle \min(1, a + c), \max(0, 1 - a - c) \rangle$
$\wedge_{1,34}$	$\langle 1 - \min(1, 2 - a - c), \min(1, 2 - a - c) \rangle$
$\vee_{1,35}$	$\langle a + c - ac, 1 - a - c + ac \rangle$
$\wedge_{1,35}$	$\langle ac, 1 - ac \rangle$
$\vee_{1,36}$	$\langle \min(1 - \min(1 - a, d), \max(a, 1 - a), \max(1 - d, d)), \max(\min(1 - a, d), \min(a, 1 - a), \min(1 - d, d)) \rangle$
$\wedge_{1,36}$	$\langle \min(\min(\max(a, c), \max((1 - a), a)), \max(c, (1 - c))), \max(\max((1 - \max(a, c)), \min((1 - a), a)), \min(c, (1 - c))) \rangle$
$\vee_{1,37}$	$\langle 1 - \min(\max(\overline{\text{sg}}(a), \overline{\text{sg}}(c)), \text{sg}(\overline{\text{sg}}(a) - \text{sg}(c))), \min(\min(\max(\overline{\text{sg}}(a), \overline{\text{sg}}(c)), \text{sg}(\overline{\text{sg}}(a) - \text{sg}(c))), \text{sg}(\overline{\text{sg}}(c) - \text{sg}(a))) \rangle$
$\wedge_{1,37}$	$\langle \text{sg}(\max(a, \text{sg}(c))\text{sg}(a + \text{sg}(c) - 1)), \overline{\text{sg}}(\max(a, \text{sg}(c))\text{sg}(a + \text{sg}(c) - 1)) \rangle$
$\vee_{1,38}$	$\langle a + (1 - a)^2(1 - d), a(1 - a) + (1 - a)^2d \rangle$
$\wedge_{1,38}$	$\langle a - a^2(1 - c), 1 - a + a^2(1 - c) \rangle$
$\vee_{1,39}$	$\langle (1 - d)\overline{\text{sg}}(a) + \text{sg}(a)(\overline{\text{sg}}(d) + (a)\text{sg}(d)), d\overline{\text{sg}}(a) + a\text{sg}(a)\text{sg}(d) \rangle$
$\wedge_{1,39}$	$\langle 1 - (1 - c)\overline{\text{sg}}(1 - a) - \text{sg}(1 - a)(\overline{\text{sg}}(c) + (1 - a)\text{sg}(c)), \rangle$

	$(1 - c)\overline{\text{sg}}(1 - a) + \text{sg}(1 - a)(\overline{\text{sg}}(c) + (1 - a)\text{sg}(c))$
$\vee_{1,40}$	$\langle 1 - \text{sg}(\overline{\text{sg}}(a) + d - 1), 1 - \overline{\text{sg}}(\overline{\text{sg}}(a) + d - 1) \rangle$
$\wedge_{1,40}$	$\langle \text{sg}(a + \text{sg}(c) - 1), \overline{\text{sg}}(a + \text{sg}(c) - 1) \rangle$
$\vee_{1,41}$	$\langle \max(\text{sg}(a), \text{sg}(c)), \min(\overline{\text{sg}}(a), \overline{\text{sg}}(c)) \rangle$
$\wedge_{1,41}$	$\langle \overline{\text{sg}}(\max(\overline{\text{sg}}(a), \overline{\text{sg}}(c))), \text{sg}(\max(\overline{\text{sg}}(a), \overline{\text{sg}}(c))) \rangle$
$\vee_{1,42}$	$\langle \max(\text{sg}(a), \text{sg}(c), \min(\overline{\text{sg}}(a), \overline{\text{sg}}(c))) \rangle$
$\wedge_{1,42}$	$\langle \overline{\text{sg}}(\max(\overline{\text{sg}}(a), \overline{\text{sg}}(c))), \text{sg}(\max(\overline{\text{sg}}(a), \overline{\text{sg}}(c))) \rangle$
$\vee_{1,43}$	$\langle \max(\text{sg}(a), 1 - d), \min(\overline{\text{sg}}(a), d) \rangle$
$\wedge_{1,43}$	$\langle \overline{\text{sg}}(\max(1 - a, 1 - c)), \max(1 - a, 1 - c) \rangle$
$\vee_{1,44}$	$\langle \max(\text{sg}(a), \text{sg}(c)), \min(\overline{\text{sg}}(a), \overline{\text{sg}}(c)) \rangle$
$\wedge_{1,44}$	$\langle \overline{\text{sg}}(\max(1 - a, 1 - c)), \max(1 - a, 1 - c) \rangle$
$\vee_{1,45}$	$\langle \max(\text{sg}(a), \text{sg}(c)), \min(\overline{\text{sg}}(a), \overline{\text{sg}}(c)) \rangle$
$\wedge_{1,45}$	$\langle \overline{\text{sg}}(\max(1 - a, \overline{\text{sg}}(c))), \max(1 - a, \overline{\text{sg}}(c)) \rangle$
$\vee_{1,46}$	$\langle \max(1 - b, \min(b, 1 - d)), \max(b, d) \rangle$
$\wedge_{1,46}$	$\langle 1 - \max(b, d), \max(b, d) \rangle$
$\vee_{1,47}$	$\langle \overline{\text{sg}}(\overline{\text{sg}}(1 - b) - \text{sg}(1 - d)), \min(\overline{\text{sg}}(1 - d), \text{sg}(\overline{\text{sg}}(1 - b) - \text{sg}(1 - d))) \rangle$
$\wedge_{1,47}$	$\langle \overline{\text{sg}}(1 - \text{sg}(c)\text{sg}(\text{sg}(c) - b)), \text{sg}(1 - \text{sg}(c)\text{sg}(\text{sg}(c) - b)) \rangle$
$\vee_{1,48}$	$\langle (1 - \min(\overline{\text{sg}}(1 - d), \text{sg}(\overline{\text{sg}}(1 - b) - \text{sg}(1 - d))), \min(\overline{\text{sg}}(1 - d), \text{sg}(\overline{\text{sg}}(1 - b) - \text{sg}(1 - d))) \rangle$
$\wedge_{1,48}$	$\langle \overline{\text{sg}}(1 - \text{sg}(1 - d)\text{sg}(\text{sg}(1 - d) - b)), \text{sg}(1 - \text{sg}(1 - d)\text{sg}(\text{sg}(1 - d) - b)) \rangle$
$\vee_{1,49}$	$\langle \min(1, 2 - b - d), \max(0, b + d - 1) \rangle$
$\wedge_{1,49}$	$\langle \max(0, 1 - b - d), 1 - \max(0, 1 - b - d) \rangle$
$\vee_{1,50}$	$\langle 1 - d, bd \rangle$
$\wedge_{1,50}$	$\langle 1 - b - d + bd, b + d - bd \rangle$
$\vee_{1,51}$	$\langle \min(\max(1 - b, c), \max(1 - b, b), \max(c, 1 - c)), \max(1 - \max(1 - b, c), \min(1 - b, b), \min(c, 1 - c)) \rangle$
$\wedge_{1,51}$	$\langle \max(\max(1 - \max(b, d), \min(1 - b, b)), \min(d, 1 - d)), 1 - \max(\max(1 - \max(b, d), \min(1 - b, b)), \min(d, 1 - d)) \rangle$
$\vee_{1,52}$	$\langle 1 - (1 - \min(\text{sg}(1 - b), c))\text{sg}(\overline{\text{sg}}(1 - b) - c), 1 - \min(\text{sg}(1 - b), c)\text{sg}(\overline{\text{sg}}(1 - b) - c) \rangle$
$\wedge_{1,52}$	$\langle \overline{\text{sg}}(\min(b, \overline{\text{sg}}(1 - d))\text{sg}(1 - b - \overline{\text{sg}}(1 - d))), \text{sg}(\min(b, \overline{\text{sg}}(1 - d))\text{sg}(1 - b - \overline{\text{sg}}(1 - d))) \rangle$
$\vee_{1,53}$	$\langle 1 - b + b^2c, (1 - b)b + b^2(1 - c) \rangle$
$\wedge_{1,53}$	$\langle (1 - b)b + (1 - b)^2(1 - d), 1 - (1 - b)b - (1 - b)^2(1 - d) \rangle$
$\vee_{1,54}$	$\langle c\overline{\text{sg}}(1 - b) + \text{sg}(1 - b)(\overline{\text{sg}}(1 - c) + (1 - b)\text{sg}(1 - c)), (1 - c)\overline{\text{sg}}(1 - b) + b\text{sg}(1 - b)\text{sg}(1 - c) \rangle$
$\wedge_{1,54}$	$\langle (1 - d)\overline{\text{sg}}(b) + (1 - b)\text{sg}(b)\text{sg}(1 - d), 1 - (1 - d)\overline{\text{sg}}(b) - (1 - b)\text{sg}(b)\text{sg}(1 - d) \rangle$
$\vee_{1,55}$	$\langle \overline{\text{sg}}(\overline{\text{sg}}(1 - b) - c), \text{sg}(\overline{\text{sg}}(1 - b) - c) \rangle$
$\wedge_{1,55}$	$\langle \text{sg}(\overline{\text{sg}}(1 - d) - b), \overline{\text{sg}}(\overline{\text{sg}}(1 - d) - b) \rangle$
$\vee_{1,56}$	$\langle \max(\text{sg}(1 - b), c), \min(\overline{\text{sg}}(1 - b), 1 - c) \rangle$

$\wedge_{1,56}$	$\langle \overline{\text{sg}}(1 - \min(\text{sg}(1 - b), \text{sg}(1 - d))), \text{sg}(1 - \min(\text{sg}(1 - b), \text{sg}(1 - d))) \rangle$
$\vee_{1,57}$	$\langle \max(\text{sg}(1 - b), \text{sg}(c)), \min(\overline{\text{sg}}(1 - b), \overline{\text{sg}}(c)) \rangle$
$\wedge_{1,57}$	$\langle \overline{\text{sg}}(1 - \min(\text{sg}(1 - b), \text{sg}(1 - d))), \text{sg}(1 - \min(\text{sg}(1 - b), \overline{\text{sg}}(1 - d))) \rangle$
$\vee_{1,58}$	$\langle \max(\overline{\text{sg}}(b), \overline{\text{sg}}(c)), 1 - \max(1 - b, \overline{\text{sg}}(c)) \rangle$
$\wedge_{1,58}$	$\langle \overline{\text{sg}}(\max(b, \overline{\text{sg}}(1 - d))), \max(b, \overline{\text{sg}}(1 - d)) \rangle$
$\vee_{1,59}$	$\langle \max(\overline{\text{sg}}(b), c), 1 - \max(1 - b, c) \rangle$
$\wedge_{1,59}$	$\langle \overline{\text{sg}}(\max(b, \overline{\text{sg}}(1 - d))), \max(b, \overline{\text{sg}}(1 - d)) \rangle$
$\vee_{1,60}$	$\langle \max(\overline{\text{sg}}(b), \overline{\text{sg}}(1 - c)), \min(b, \text{sg}(c)) \rangle$
$\wedge_{1,60}$	$\langle \overline{\text{sg}}(1 - \min(1 - b, \text{sg}(1 - d))), 1 - \min(1 - b, \text{sg}(1 - d)) \rangle$
$\vee_{1,61}$	$\langle \max(c, \min(a, d)), \min(b, d) \rangle$
$\wedge_{1,61}$	$\langle \min(a, c), \max(d, \min(b, c)) \rangle$
$\vee_{1,62}$	$\langle \overline{\text{sg}}(d - a), \overline{\text{sg}}(1 - b)\text{sg}(d - a) \rangle$
$\wedge_{1,62}$	$\langle \overline{\text{sg}}(1 - \text{asg}(c - b)), \overline{\text{sg}}(c - b) \rangle$
$\vee_{1,63}$	$\langle 1 - (1 - a)\text{sg}(d - a), \text{bsg}(d - a) \rangle$
$\wedge_{1,63}$	$\langle \text{asg}(c - b), 1 - (1 - b)\text{sg}(c - b) \rangle$
$\vee_{1,64}$	$\langle c + ad, bd \rangle$
$\wedge_{1,64}$	$\langle ac, d + bc \rangle$
$\vee_{1,65}$	$\langle 1 - (1 - \min(\text{sg}(a), c))\text{sg}(d - \text{sg}(a)), \max(\overline{\text{sg}}(1 - b), d)\text{sg}(d - \text{sg}(a))\text{sg}(\overline{\text{sg}}(1 - b) - c) \rangle$
$\wedge_{1,65}$	$\langle \overline{\text{sg}}(1 - \max(a, \text{sg}(c))\text{sg}(\text{sg}(c) - b)\text{sg}(a - \overline{\text{sg}}(1 - d))), \text{sg}(1 - (1 - \min(b, \overline{\text{sg}}(1 - d)))\text{sg}(\text{sg}(c) - b)) \rangle$
$\vee_{1,66}$	$\langle c + d^2a, ad + d^2b \rangle$
$\wedge_{1,66}$	$\langle bc + c^2a, d + c^2b \rangle$
$\vee_{1,67}$	$\langle a\overline{\text{sg}}(1 - d) + \text{sg}(1 - d)(\overline{\text{sg}}(1 - a) + \text{csg}(1 - a)), b\overline{\text{sg}}(1 - d) + d\text{sg}(1 - d)\text{sg}(1 - a) \rangle$
$\wedge_{1,67}$	$\langle a\overline{\text{sg}}(1 - c) + d\text{sg}(1 - c)\text{sg}(1 - b), b\overline{\text{sg}}(1 - c) + \text{sg}(1 - c)(\overline{\text{sg}}(1 - b) + d\text{sg}(1 - b)) \rangle$
$\vee_{1,68}$	$\langle 1 - (1 - a)\text{sg}(d - a), \text{bsg}(d - a)\text{sg}(b - c) \rangle$
$\wedge_{1,68}$	$\langle \text{asg}(c - b)\text{sg}(a - d), 1 - (1 - b)\text{sg}(c - b) \rangle$
$\vee_{1,69}$	$\langle 1 - (1 - a)\text{sg}(d - a) - b\overline{\text{sg}}(d - a)\text{sg}(b - c), \text{bsg}(b - c) \rangle$
$\wedge_{1,69}$	$\langle \text{asg}(a - d), 1 - (1 - b)\text{sg}(c - b) - a\overline{\text{sg}}(c - b)\text{sg}(a - d) \rangle$
$\vee_{1,70}$	$\langle \max(\overline{\text{sg}}(d), a), \min(\text{sg}(d), b) \rangle$
$\wedge_{1,70}$	$\langle \min(\text{sg}(c), a), \max(\overline{\text{sg}}(c), b) \rangle$
$\vee_{1,71}$	$\langle \max(c, a), \min(dc + d^2, b) \rangle$
$\wedge_{1,71}$	$\langle \min(cd + c^2, a), \max(b, d) \rangle$
$\vee_{1,72}$	$\langle \max(a, c), \min(1 - c, b) \rangle$
$\wedge_{1,72}$	$\langle \min(1 - d, a), \max(b, d) \rangle$
$\vee_{1,73}$	$\langle \max(1 - \max(\text{sg}(d), \text{sg}(1 - c)), a), \min(\text{sg}(1 - c), b) \rangle$
$\wedge_{1,73}$	$\langle \min(\text{sg}(1 - d), a), \max(1 - \max(\text{sg}(c), \text{sg}(1 - d)), b) \rangle$
$\vee_{1,74}$	$\langle \max(\overline{\text{sg}}(d), \overline{\text{sg}}(b), \min(\text{sg}(d), \text{sg}(b))) \rangle$

$\wedge_{1,74}$	$\langle \text{sg}(\max(\overline{\text{sg}}(d), \overline{\text{sg}}(b))), \overline{\text{sg}}(\max(\overline{\text{sg}}(d), \overline{\text{sg}}(b))) \rangle$
$\vee_{1,75}$	$\langle \max(c, (a^2b + ab^2 + a^2)((b^2 + ab) + (a^2b + ab^2 + a^2))), \min(d(c + d), (b^2 + ab)((a^2b + ab^2 + a^2)^2 + (b^2 + ab) + (b^2 + ab)(a^2b + ab^2 + a^2)) \rangle$
$\wedge_{1,75}$	$\langle \min((\min(1, c(d^2 + c + dc)))(\min(1, c(d^2 + c + dc)) + d(d + c)), (a(b^2 + a + ba)))(\min((\min(1, c(d^2 + c + dc)))(\min(1, c(d^2 + c + dc)) + d(d + c))), a((b^2 + a + ba) + \max(d(d + c), b(b + a))), \min(1, (\max(d(d + c), b(b + a))((\min((\min(1, c((d^2 + c) + dc)) \min(1, (c((d^2 + c) + dc))) + d(d + c))), a((b^2 + a + ba))^2 + \max(d(d + c), b(b + a))) + (\min((\min(1, c((d^2 + c) + dc)))(\min(1, c((d^2 + c) + dc)) + d(d + c))), (a(b^2 + a + ba)) \max(d(d + c), b(b + a)))))) \rangle$
$\vee_{1,76}$	$\langle \max(c, a), 1 - \max(c, a) \rangle$
$\wedge_{1,76}$	$\langle \min(a, c), 1 - \min(a, c) \rangle$
$\vee_{1,77}$	$\langle 1 - \min(\text{sg}(1 - c), \text{sg}(1 - a)), \min(\text{sg}(1 - c), \text{sg}(1 - a)) \rangle$
$\wedge_{1,77}$	$\langle \text{sg}(\min(\overline{\text{sg}}(1 - c), \overline{\text{sg}}(1 - a))), \overline{\text{sg}}(\min(\overline{\text{sg}}(1 - c), \overline{\text{sg}}(1 - a))) \rangle$
$\vee_{1,78}$	$\langle \max(\overline{\text{sg}}(1 - c), a), \min(\text{sg}(d), b) \rangle$
$\wedge_{1,78}$	$\langle \min(\text{sg}(c), a), \max(\overline{\text{sg}}(1 - d), b) \rangle$
$\vee_{1,79}$	$\langle \max(\overline{\text{sg}}(1 - c), \overline{\text{sg}}(1 - a)), \min(\text{sg}(d), \text{sg}(b)) \rangle$
$\wedge_{1,79}$	$\langle \text{sg}(\min(\overline{\text{sg}}(1 - c), \overline{\text{sg}}(1 - a))), \overline{\text{sg}}(1 - \max(\text{sg}(d), \text{sg}(b))) \rangle$
$\vee_{1,80}$	$\langle \max(\overline{\text{sg}}(1 - c), a), \min(d, b) \rangle$
$\wedge_{1,80}$	$\langle \min(c, a), \max(\overline{\text{sg}}(1 - d), b) \rangle$
$\vee_{1,81}$	$\langle \max(\overline{\text{sg}}(1 - a), \overline{\text{sg}}(1 - c)), \min(d, \overline{\text{sg}}(1 - b)) \rangle$
$\wedge_{1,81}$	$\langle \overline{\text{sg}}(1 - \min(\overline{\text{sg}}(1 - c), \overline{\text{sg}}(1 - a))), \overline{\text{sg}}(1 - \max(\overline{\text{sg}}(1 - b), \overline{\text{sg}}(1 - d))) \rangle$
$\vee_{1,82}$	$\langle \max(1 - d, \min(d, a)), \min(d, 1 - a) \rangle$
$\wedge_{1,82}$	$\langle 1 - \max(1 - c, \min(c, 1 - a)), \max(1 - c, \min(c, 1 - a)) \rangle$
$\vee_{1,83}$	$\langle \overline{\text{sg}}(\overline{\text{sg}}(a) + d - 1), \overline{\text{sg}}(a)\text{sg}(\overline{\text{sg}}(a) + d - 1) \rangle$
$\wedge_{1,83}$	$\langle \text{sg}((a + c - 1)), \overline{\text{sg}}((a + c - 1)) \rangle$
$\vee_{1,84}$	$\langle 1 - (1 - a)\text{sg}(d - a), (1 - a)\text{sg}(d - a) \rangle$
$\wedge_{1,84}$	$\langle a\text{sg}(c + a - 1), 1 - a\text{sg}(c + a - 1) \rangle$
$\vee_{1,85}$	$\langle 1 - d + d^2a, d(1 - d) + d^2 \rangle$
$\wedge_{1,85}$	$\langle c - c^2(1 - a), 1 - c + c^2(1 - a) \rangle$
$\vee_{1,86}$	$\langle a\overline{\text{sg}}(1 - d) + \text{sg}(1 - d)(\overline{\text{sg}}(1 - a) + (1 - d)\text{sg}(d)), (1 - a)\overline{\text{sg}}(1 - d) + d\text{sg}(1 - d)\text{sg}(1 - a) \rangle$
$\wedge_{1,86}$	$\langle 1 - ((1 - a)\overline{\text{sg}}(1 - c) + \text{sg}(1 - c)(\overline{\text{sg}}(a) + (1 - c)\text{sg}(c))), (1 - a)\overline{\text{sg}}(1 - c) + \text{sg}(1 - c)(\overline{\text{sg}}(a) + (1 - c)\text{sg}(c)) \rangle$
$\vee_{1,87}$	$\langle \max(\overline{\text{sg}}(d), a), \min(\text{sg}(d), 1 - a) \rangle$
$\wedge_{1,87}$	$\langle 1 - \max(\overline{\text{sg}}(c), 1 - a), \max(\overline{\text{sg}}(c), 1 - a) \rangle$
$\vee_{1,88}$	$\langle \max(\overline{\text{sg}}(d), \overline{\text{sg}}(1 - a)), \min(\text{sg}(d), \text{sg}(1 - a)) \rangle$
$\wedge_{1,88}$	$\langle \text{sg}(1 - \max(\text{sg}(1 - c), \text{sg}(1 - a))), \overline{\text{sg}}(1 - \max(\text{sg}(1 - c), \text{sg}(1 - a))) \rangle$

$\vee_{1,89}$	$\langle \max(\overline{\text{sg}}(d), a), \min(d, 1 - a) \rangle$
$\wedge_{1,89}$	$\langle 1 - \max(\overline{\text{sg}}(c), 1 - a), \max(\overline{\text{sg}}(c), 1 - a) \rangle$
$\vee_{1,90}$	$\langle \max(\text{sg}(a), \overline{\text{sg}}(d)), \min(d, \overline{\text{sg}}(a)) \rangle$
$\wedge_{1,90}$	$\langle \overline{\text{sg}}(\max(\overline{\text{sg}}(a), \text{sg}(1 - c))), \overline{\text{sg}}(1 - \max(\overline{\text{sg}}(a), \text{sg}(1 - c))) \rangle$
$\vee_{1,91}$	$\langle \max(c, \min(1 - c, 1 - b)), 1 - \max(1 - b, c) \rangle$
$\wedge_{1,91}$	$\langle 1 - \max(b, d), \max(b, d) \rangle$
$\vee_{1,92}$	$\langle \overline{\text{sg}}(b - c), \min(b, \text{sg}(b - c)) \rangle$
$\wedge_{1,92}$	$\langle \overline{\text{sg}}(1 - \min(1 - b, \text{sg}(\text{sg}(1 - d) - b))), 1 - \min(1 - b, \text{sg}(\text{sg}(1 - d) - b)) \rangle$
$\vee_{1,93}$	$\langle (1 - \min(b, \text{sg}(b - c))), \min(b, \text{sg}(b - c)) \rangle$
$\wedge_{1,93}$	$\langle \min(1 - b, \text{sg}(1 - b - d)), 1 - \min(1 - b, \text{sg}(1 - b - d)) \rangle$
$\vee_{1,94}$	$\langle c + (1 - c)^2(1 - b), (1 - c)c + (1 - c)^2b \rangle$
$\wedge_{1,94}$	$\langle (1 - d)d + (1 - d)^2(1 - b), 1 - (1 - d)d - (1 - d)^2(1 - b) \rangle$
$\vee_{1,95}$	$\langle \min(1 - b, \overline{\text{sg}}(c)) + \text{sg}(c)(\overline{\text{sg}}(b) + \min(c, \text{sg}(b))),$ $\min(b, \overline{\text{sg}}(c)) + \min(1 - c, \text{sg}(c), \text{sg}(b)) \rangle$
$\wedge_{1,95}$	$\langle \min(1 - b, \overline{\text{sg}}(d)) + \min(1 - d, \text{sg}(d), \text{sg}(1 - b)),$ $1 - \min(1 - b, \overline{\text{sg}}(d)) - \min(1 - d, \text{sg}(d), \text{sg}(1 - b)) \rangle$
$\vee_{1,96}$	$\langle \max(\overline{\text{sg}}(1 - c), 1 - b), \min(\text{sg}(b), 1 - c) \rangle$
$\wedge_{1,96}$	$\langle \min(\text{sg}(1 - b), 1 - d), 1 - \min(\text{sg}(1 - b), 1 - d) \rangle$
$\vee_{1,97}$	$\langle \max(\overline{\text{sg}}(1 - c), \overline{\text{sg}}(b)), \min(\text{sg}(1 - c), \text{sg}(b)) \rangle$
$\wedge_{1,97}$	$\langle \text{sg}(\min(\overline{\text{sg}}(d), \overline{\text{sg}}(b))), \overline{\text{sg}}(\min(\overline{\text{sg}}(d), \overline{\text{sg}}(b))) \rangle$
$\vee_{1,98}$	$\langle \max(\overline{\text{sg}}(1 - c), 1 - b), 1 - \max(1 - b, c) \rangle$
$\wedge_{1,98}$	$\langle 1 - \max(b, d), \max(b, d) \rangle$
$\vee_{1,99}$	$\langle \max(\overline{\text{sg}}(1 - c), \overline{\text{sg}}(b)), \min(1 - c, \text{sg}(b)) \rangle$
$\wedge_{1,99}$	$\langle \overline{\text{sg}}(1 - \min(\text{sg}(1 - d), \overline{\text{sg}}(b))), \overline{\text{sg}}(\min(\text{sg}(1 - d), \overline{\text{sg}}(b))) \rangle$
$\vee_{1,100}$	$\langle \max(\min(\text{asg}(b), \text{sg}(\text{bsg}(a))), c), \min(\text{bsg}(a), \text{sg}(\text{asg}(b)), d) \rangle$
$\wedge_{1,100}$	$\langle \min(a, \text{sg}(b), c\overline{\text{sg}}(d))\text{sg}(\max(\min(b, \text{sg}(a)), d\overline{\text{sg}}(c))),$ $\max(\min(b, \text{sg}(a)), d\overline{\text{sg}}(c))\text{sg}(\min(a, \text{sg}(b), c\overline{\text{sg}}(d))) \rangle$
$\vee_{1,101}$	$\langle \min(c, \text{sg}(d)), 0 \rangle$
$\wedge_{1,101}$	$\langle 0, 0 \rangle$
$\vee_{1,102}$	$\langle \min(c, \text{sg}(d)), \min(b, \min(\text{sg}(c), d)) \rangle$
$\wedge_{1,102}$	$\langle 0, 0 \rangle$
$\vee_{1,103}$	$\langle \max(\min(1 - \min((1 - a), \text{sg}(a))), \text{sg}(\min((1 - a),$ $\text{sg}(a))), 1 - d), \min(\min((1 - a), \text{sg}(a)),$ $\text{sg}(1 - \min((1 - a), \text{sg}(a))), d) \rangle$
$\wedge_{1,103}$	$\langle \min((1 - \max(\min((1 - a), \text{sg}(a)),$ $(1 - \min(c, \text{sg}(1 - c))))), \max(\min(\text{sg}(1 - a),$ $\text{sg}(a)), \text{sg}(1 - \min(c, \text{sg}(1 - c))))),$ $\min(\max(\min((1 - a), \text{sg}(a)), (1$ $- \min(c, \text{sg}(1 - c))), \text{sg}(1 - \max(\min((1 - a),$ $\text{sg}(a)), (1 - \min(c, \text{sg}(1 - c)))))) \rangle$
$\vee_{1,104}$	$\langle \max(\min(1 - \min((1 - a), \text{sg}(a))),$ $\text{sg}(\min((1 - a), \text{sg}(a))),$

	$\langle \min(1 - d, \text{sg}(d)), \min(\min((1 - a), \text{sg}(a)), \text{sg}(1 - \min((1 - a), \text{sg}(a))), d, \text{sg}(1 - d)) \rangle$
$\wedge_{1,104}$	$\langle \min(1 - \min(1 - a, \text{sg}(a)), \text{sg}(\min(1 - a, \text{sg}(a))), 0 \rangle$
$\vee_{1,105}$	$\langle \max(a, \min(1 - d, \text{sg}(d))), \min(1 - a, d, \text{sg}(1 - d)) \rangle$
$\wedge_{1,105}$	$\langle a, 0 \rangle$
$\vee_{1,106}$	$\langle \max(\min(\min(1 - b, \text{sg}(b)), \text{sg}(1 - \min(1 - b, \text{sg}(b))), c), \min(1 - \min(1 - b, \text{sg}(b)), \text{sg}(\min(1 - b, \text{sg}(b))), 1 - c) \rangle$
$\wedge_{1,106}$	$\langle \min(1 - b, \text{sg}(b), 1 - \min(d, \text{sg}(1 - d))), \text{sg}(1 - \min(1 - b, \text{sg}(b), 1 - \min(d, \text{sg}(1 - d)))) \rangle$
$\vee_{1,107}$	$\langle c \min(c, \text{sg}(1 - c)), 0 \rangle$
$\wedge_{1,107}$	$\langle \min(\min(1 - b, \text{sg}(b), 1 - \min(d, \text{sg}(1 - d))), \text{sg}(\min(d, \text{sg}(1 - d))), \text{sg}(1 - \min(1 - b, \text{sg}(b), 1 - \min(d, \text{sg}(1 - d))), \text{sg}(\min(d, \text{sg}(1 - d)))) \rangle, 0 \rangle$
$\vee_{1,108}$	$\langle \min(c, \text{sg}(1 - c)), \min(1 - c, \text{sg}(c)) \rangle$
$\wedge_{1,108}$	$\langle \min(1 - b, 1 - d, \text{sg}(d)), 0 \rangle$
$\vee_{1,109}$	$\langle (ab + \overline{\text{sg}}(1 - a)) + \min(\overline{\text{sg}}(1 - b), c), (ab + \overline{\text{sg}}(1 - a))b + \min(\overline{\text{sg}}(1 - b), d) \rangle$
$\wedge_{1,109}$	$\langle \min(ab + \overline{\text{sg}}(1 - a), cd + \overline{\text{sg}}(1 - c)), \max(b, d) \min(ab + \overline{\text{sg}}(1 - a), cd + \overline{\text{sg}}(1 - c)) + \overline{\text{sg}}(1 - \max(b, d)) \rangle$
$\vee_{1,110}$	$\langle \max((ab + \overline{\text{sg}}(1 - a)), c), \min((ab + \overline{\text{sg}}(1 - a))b + \overline{\text{sg}}(1 - b), d) \rangle$
$\wedge_{1,110}$	$\langle \min((ab + \overline{\text{sg}}(1 - a))((ab + \overline{\text{sg}}(1 - a))b + \overline{\text{sg}}(1 - b)) + \overline{\text{sg}}(1 - (ab + \overline{\text{sg}}(1 - a))), (cd + \overline{\text{sg}}(1 - c))), \max(((ab + \overline{\text{sg}}(1 - a))b + \overline{\text{sg}}(1 - b)), d) \cdot \min((ab + \overline{\text{sg}}(1 - a))((ab + \overline{\text{sg}}(1 - a))b + \overline{\text{sg}}(1 - b)) + \overline{\text{sg}}(1 - (ab + \overline{\text{sg}}(1 - a))), (cd + \overline{\text{sg}}(1 - c))) + \overline{\text{sg}}(1 - \max(((ab + \overline{\text{sg}}(1 - a))b + \overline{\text{sg}}(1 - b)), d)) \rangle$
$\vee_{1,111}$	$\langle \max((ab + \overline{\text{sg}}(1 - a)), cd + \overline{\text{sg}}(1 - c)), \min(((ab + \overline{\text{sg}}(1 - a))b + \overline{\text{sg}}(1 - b)), d(cd + \overline{\text{sg}}(1 - c)) + \overline{\text{sg}}(1 - d)) \rangle$
$\wedge_{1,111}$	$\langle \min(ab + \overline{\text{sg}}(1 - a), (cd + \overline{\text{sg}}(1 - c))(d(cd + \overline{\text{sg}}(1 - c)) + \overline{\text{sg}}(1 - d)) + \overline{\text{sg}}(1 - d)), \max(b, d(cd + \overline{\text{sg}}(1 - c)) + \overline{\text{sg}}(1 - d)) \cdot \min(ab + \overline{\text{sg}}(1 - a), (cd + \overline{\text{sg}}(1 - c))(d(cd + \overline{\text{sg}}(1 - c)) + \overline{\text{sg}}(1 - d)) + \overline{\text{sg}}(1 - d)) \rangle$
$\vee_{1,112}$	$\langle ((ab + \overline{\text{sg}}(1 - a)) + c - (ab + \overline{\text{sg}}(1 - a))c, (ab + \overline{\text{sg}}(1 - a))b + \overline{\text{sg}}(1 - b)d) \rangle$
$\wedge_{1,112}$	$\langle ab + \overline{\text{sg}}(1 - a)(cd + \overline{\text{sg}}(1 - c)), (b + d - bd)((ab + \overline{\text{sg}}(1 - a)(cd + \overline{\text{sg}}(1 - c))) + \overline{\text{sg}}(1 - b - d + bd)) \rangle$
$\vee_{1,113}$	$\langle (ab + \overline{\text{sg}}(1 - a)) + cd - (ab + \overline{\text{sg}}(1 - a))(cd + \overline{\text{sg}}(1 - c)), \rangle$

	$((ab + \overline{sg}(1-a))b + \overline{sg}(1-b))(d(cd + \overline{sg}(1-c)) + \overline{sg}(1-d))$
$\wedge_{1,113}$	$\langle (ab + \overline{sg}(1-a))(cd + \overline{sg}(1-c))(cd + \overline{sg}(1-c))d + \overline{sg}(1-d) + \overline{sg}(sg(1-c) - cd), (ab + \overline{sg}(1-a))(cd + \overline{sg}(1-c))(cd + \overline{sg}(1-c))d + \overline{sg}(1-d) + \overline{sg}(sg(1-c) - cd) \cdot (b + (cd + \overline{sg}(1-c))d - b((cd + \overline{sg}(1-c))d + \overline{sg}(1-d))) + \overline{sg}(1 - (b + (cd + \overline{sg}(1-c))d - b((cd + \overline{sg}(1-c))d + \overline{sg}(1-d)))) \rangle$
$\vee_{1,114}$	$\langle a + \min(\overline{sg}(a), 1-d), a(1-a) + \min(\overline{sg}(a), d) \rangle$
$\wedge_{1,114}$	$\langle a - \min(\overline{sg}(1-a), sg(1-c) - c(1-c)), (1-a - \min(\overline{sg}(1-a), sg(1-c) - c(1-c))) (a - \min(\overline{sg}(1-a), sg(1-c) - c(1-c))) + \overline{sg}((a - \min(\overline{sg}(1-a), sg(1-c) - c(1-c)))) \rangle$
$\vee_{1,115}$	$\langle 1 - \min(1-a, d), \min(a(1-a) + \overline{sg}(a), d) \rangle$
$\wedge_{1,115}$	$\langle \min(1-a, c(1-c) + \overline{sg}(1-c)), (1 - \min(a, c(1-c) + \overline{sg}(1-c))) \min(a, c(1-c) + \overline{sg}(1-c)) + \overline{sg}(\min(a, c(1-c) + \overline{sg}(1-c))) \rangle$
$\vee_{1,116}$	$\langle \max(a, (1-d)d + \overline{sg}(d)), \min(a(1-a) + \overline{sg}(a), d((1-d)d + \overline{sg}(d)) + \overline{sg}(1-d)) \rangle$
$\wedge_{1,116}$	$\langle 1 - \max(1-a, (1-c(1-c) - \overline{sg}(1-c)))(c(1-c) + \overline{sg}(1-c)) + \overline{sg}(c(1-c) + \overline{sg}(1-c)), (1 - \max(1-a, (1-c(1-c) - \overline{sg}(1-c)))(c(1-c) + \overline{sg}(1-c)) + \overline{sg}(c(1-c) + \overline{sg}(1-c))) \max(1-a, (1-c(1-c) - \overline{sg}(1-c)) (c(1-c) + \overline{sg}(1-c)) + \overline{sg}(c(1-c) + \overline{sg}(1-c))) \cdot \overline{sg}(1 - \max(1-a, (1-c(1-c) - \overline{sg}(1-c)))(c(1-c) + \overline{sg}(1-c)) + \overline{sg}(c(1-c) + \overline{sg}(1-c))) \rangle$
$\vee_{1,117}$	$\langle a - ad, (a(1-a) + \overline{sg}(a))d \rangle$
$\wedge_{1,117}$	$\langle 1 - (1-a)(sg(1-c) - c(1-c)), (1-a)(sg(1-c) - c(1-c))(1 - (1-a)(sg(1-c) - c(1-c))) + \overline{sg}(1 - (1-a)(sg(1-c) - c(1-c))) \rangle$
$\vee_{1,118}$	$\langle a + (1-d)d - a((1-d)d + \overline{sg}(d)), (a(1-a) + \overline{sg}(a))d((1-d)d + \overline{sg}(d)) + \overline{sg}(1-d) \rangle$
$\wedge_{1,118}$	$\langle 1 - (1-a + (1 - (c(1-c) + \overline{sg}(1-c)))(c(1-c) + \overline{sg}(1-c)) - (1-a)((1 - (c(1-c) + \overline{sg}(1-c)))(c(1-c) + \overline{sg}(1-c)) + \overline{sg}((c(1-c) + \overline{sg}(1-c))))) , (1-a + (1 - (c(1-c) + \overline{sg}(1-c)))(c(1-c) + \overline{sg}(1-c)) - (1-a)((1 - (c(1-c) + \overline{sg}(1-c)))(c(1-c) + \overline{sg}(1-c)) + \overline{sg}((c(1-c) + \overline{sg}(1-c))))) \cdot (1 - (1-a + (1 - (c(1-c) + \overline{sg}(1-c)))(c(1-c) + \overline{sg}(1-c)) - (1-a)((1 - (c(1-c) + \overline{sg}(1-c)))(c(1-c) + \overline{sg}(1-c))))) \rangle$

	$\langle (c(1-c) + \overline{\text{sg}}(1-c)) + \overline{\text{sg}}((c(1-c) + \overline{\text{sg}}(1-c))) \rangle$ $+ \overline{\text{sg}}(1 - (1-a + (1 - (c(1-c) + \overline{\text{sg}}(1-c))))(c(1-c)$ $+ \overline{\text{sg}}(1-c)) - (1-a)((1 - (c(1-c) + \overline{\text{sg}}(1-c)))$ $\cdot (c(1-c) + \overline{\text{sg}}(1-c)) + \overline{\text{sg}}((c(1-c) + \overline{\text{sg}}(1-c)))) \rangle$
$\vee_{1,119}$	$\langle b(1-b) + \overline{\text{sg}}(b) + \min(\overline{\text{sg}}(b(1-b) + \overline{\text{sg}}(b)), c),$ $(\overline{\text{sg}}(b - b(1-b)))(b(1-b) + \overline{\text{sg}}(b)) + \min(\overline{\text{sg}}(b(1-b)$ $+ \overline{\text{sg}}(b)), 1-c) \rangle$
$\wedge_{1,119}$	$\langle (1-b)b + \min(\overline{\text{sg}}(b), (1-d)),$ $((1 - (((1-b)b) + \min(\overline{\text{sg}}(b), (1-d))))((1-b)b$ $+ \min(\overline{\text{sg}}(b), (1-d)))) + \overline{\text{sg}}(((1-b)b) + \min(\overline{\text{sg}}(b),$ $(1-d))) \rangle$
$\vee_{1,120}$	$\langle \max((b(1-b) + \overline{\text{sg}}(b)), c),$ $\min((1 - (b(1-b) + \overline{\text{sg}}(b)))(b(1-b)$ $+ \overline{\text{sg}}(b)) + \overline{\text{sg}}((b(1-b) + \overline{\text{sg}}(b))), 1-c) \rangle$
$\wedge_{1,120}$	$\langle \min((1-b)b + \overline{\text{sg}}(b), 1-d),$ $(1 - \min((1-b)b + \overline{\text{sg}}(b), 1-d)) \min((1-b)b$ $+ \overline{\text{sg}}(b), 1-d) + \overline{\text{sg}}(\min((1-b)b + \overline{\text{sg}}(b), 1-d)) \rangle$
$\vee_{1,121}$	$\langle \max((b(1-b) + \overline{\text{sg}}(b)), c(1-c) + \overline{\text{sg}}(1-c)),$ $\min((1 - (b(1-b) + \overline{\text{sg}}(b)))(b(1-b) + \overline{\text{sg}}(b))$ $+ \overline{\text{sg}}((b(1-b) + \overline{\text{sg}}(b))), (1-c)(c(1-c) + \overline{\text{sg}}(1-c))) + \overline{\text{sg}}(c) \rangle$
$\wedge_{1,121}$	$\langle \min((1-b)b + \overline{\text{sg}}(b), (1-c)(c(1-c) + \overline{\text{sg}}(1-c)))$ $+ \overline{\text{sg}}(c), (\min((1-b)b + \overline{\text{sg}}(b), (1-c)(c(1-c)$ $+ \overline{\text{sg}}(1-c))) + \overline{\text{sg}}(c)) \cdot (1 - (\min((1-b)b + \overline{\text{sg}}(b),$ $(1-c)(c(1-c) + \overline{\text{sg}}(1-c))) + \overline{\text{sg}}(c)))$ $+ \overline{\text{sg}}(\min((1-b)b + \overline{\text{sg}}(b), (1-c)(c(1-c) + \overline{\text{sg}}(1-c))) + \overline{\text{sg}}(c)) \rangle$
$\vee_{1,122}$	$\langle (b(1-b) + \overline{\text{sg}}(b)) + c - (b(1-b) + \overline{\text{sg}}(b))c,$ $((1-c)(b(1-b) + \overline{\text{sg}}(b)) + \overline{\text{sg}}((b(1-b) + \overline{\text{sg}}(b))))(1-c) \rangle$
$\wedge_{1,122}$	$\langle (((1-d)b) + \overline{\text{sg}}(b))(1-d),$ $(1 - (((1-d)b) + \overline{\text{sg}}(b))(1-d))(((1-d)b$ $+ \overline{\text{sg}}(b))(1-d) + \overline{\text{sg}}(((1-d)b) + \overline{\text{sg}}(b))(1-d)) \rangle$
$\vee_{1,123}$	$\langle b(1-b) + \overline{\text{sg}}(b) + c(1-c) - ((b(1-b) + \overline{\text{sg}}(b))(c(1-c)$ $+ \overline{\text{sg}}(1-c))), ((1 - (b(1-b) + \overline{\text{sg}}(b)))(b(1-b) + \overline{\text{sg}}(b))$ $+ \overline{\text{sg}}((b(1-b) + \overline{\text{sg}}(b))))((1-c)(c(1-c) + \overline{\text{sg}}(1-c)))$ $+ \overline{\text{sg}}(c) \rangle$
$\wedge_{1,123}$	$\langle ((1-b)b + \overline{\text{sg}}(b))(((1-d)(d(1-d) + \overline{\text{sg}}(1-d)))$ $+ \overline{\text{sg}}(d)), (1 - ((1-b)b + \overline{\text{sg}}(b))(((1-d)(d(1-d)$ $+ \overline{\text{sg}}(1-d))) + \overline{\text{sg}}(d))((1-b)b + \overline{\text{sg}}(b))$ $\cdot (((1-d)(d(1-d) + \overline{\text{sg}}(1-d))) + \overline{\text{sg}}(d))$ $+ \overline{\text{sg}}(((1-b)b + \overline{\text{sg}}(b))(((1-d)(d(1-d)$ $+ \overline{\text{sg}}(1-d))) + \overline{\text{sg}}(d))) \rangle$
$\vee_{1,124}$	$\langle c + \min(\overline{\text{sg}}(1-d), a), cd + \min(\overline{\text{sg}}(1-d), b) \rangle$
$\wedge_{1,124}$	$\langle cd + \min(\overline{\text{sg}}(1-c), a), d + \min(\overline{\text{sg}}(1-c), b) \rangle$
$\vee_{1,125}$	$\langle \max(a, c), \min(cd + \overline{\text{sg}}(1-d), b) \rangle$

$\wedge_{1,125}$	$\langle \min(cd + \overline{\text{sg}}(1 - c), a), \max(b, d) \rangle$
$\vee_{1,126}$	$\langle \max(c, (ab + \overline{\text{sg}}(1 - b))(a(ab + \overline{\text{sg}}(1 - b)) + \overline{\text{sg}}(1 - a)) + \overline{\text{sg}}(1 - a(ab + \overline{\text{sg}}(1 - b)) - \overline{\text{sg}}(1 - a))), \min(cd + \overline{\text{sg}}(1 - d), (ab + \overline{\text{sg}}(1 - b))((ab + \overline{\text{sg}}(1 - b)) \cdot (a(ab + \overline{\text{sg}}(1 - b)) + \overline{\text{sg}}(1 - a)) + \overline{\text{sg}}(1 - a(ab + \overline{\text{sg}}(1 - b)) - \overline{\text{sg}}(1 - a))) + \overline{\text{sg}}(1 - (ab + \overline{\text{sg}}(1 - b)))) \rangle$
$\wedge_{1,126}$	$\langle \min(cd + \overline{\text{sg}}(1 - d), (ab + \overline{\text{sg}}(1 - b))((ab + \overline{\text{sg}}(1 - b))a(ab + \overline{\text{sg}}(1 - b) + \overline{\text{sg}}(1 - a)) + \overline{\text{sg}}(1 - a(ab + \overline{\text{sg}}(1 - b) + \overline{\text{sg}}(1 - a)))) + \overline{\text{sg}}(1 - (ab + \overline{\text{sg}}(1 - b))))), \max(c, (ab + \overline{\text{sg}}(1 - b))a(ab + \overline{\text{sg}}(1 - b) + \overline{\text{sg}}(1 - a)) + \overline{\text{sg}}(1 - a(ab + \overline{\text{sg}}(1 - b) + \overline{\text{sg}}(1 - a)))) \rangle$
$\vee_{1,127}$	$\langle a + c - ac, (cd + \overline{\text{sg}}(1 - d))b \rangle$
$\wedge_{1,127}$	$\langle (cd + \overline{\text{sg}}(1 - c))a, b + d - bd \rangle$
$\vee_{1,128}$	$\langle c + ab(a(ab + \overline{\text{sg}}(1 - b)) + \overline{\text{sg}}(1 - a)) - c(ab(a(ab + \overline{\text{sg}}(1 - b)) + \overline{\text{sg}}(1 - b)) + \overline{\text{sg}}(1 - a)) + \overline{\text{sg}}(1 - (a(ab + \overline{\text{sg}}(1 - b)) + \overline{\text{sg}}(1 - a))), (cd + \overline{\text{sg}}(1 - d))(ab(ab(a(ab + \overline{\text{sg}}(1 - b)) + \overline{\text{sg}}(1 - a)) + \overline{\text{sg}}(1 - (a(ab + \overline{\text{sg}}(1 - b)) + \overline{\text{sg}}(1 - a)))) + \overline{\text{sg}}(1 - ab) \rangle$
$\wedge_{1,128}$	$\langle ((cd + ab - cd(ab + \overline{\text{sg}}(1 - b)))(cd(c^2d + c\overline{\text{sg}}(1 - d) + \overline{\text{sg}}(1 - c)) + \overline{\text{sg}}(\text{sg}(1 - c) - c^2d - c\overline{\text{sg}}(1 - d)))(a(ab + \overline{\text{sg}}(1 - b)) + \overline{\text{sg}}(1 - a)), (cd + ab - cd(ab + \overline{\text{sg}}(1 - b)))(cd(c^2d + c\overline{\text{sg}}(1 - d) + \overline{\text{sg}}(1 - c)) + \overline{\text{sg}}(\text{sg}(1 - c) - c^2d - c\overline{\text{sg}}(1 - d)))(a(ab + \overline{\text{sg}}(1 - b)) + \overline{\text{sg}}(1 - a)) + \overline{\text{sg}}(1 - (cd(c^2d + c\overline{\text{sg}}(1 - d) + \overline{\text{sg}}(1 - c)) + \overline{\text{sg}}(\text{sg}(1 - c) - c^2d - c\overline{\text{sg}}(1 - d)))(a(ab + \overline{\text{sg}}(1 - b)) + \overline{\text{sg}}(1 - a))) + \overline{\text{sg}}(1 - cd - ab + cd(ab + \overline{\text{sg}}(1 - b)))) \rangle$
$\vee_{1,129}$	$\langle 1 - d + \min(\overline{\text{sg}}(1 - d), a), d(1 - d) + \min(\overline{\text{sg}}(1 - d), 1 - a) \rangle$
$\wedge_{1,129}$	$\langle c - \min(\overline{\text{sg}}(1 - c), 1 - a), 1 - c + \min(\overline{\text{sg}}(1 - c), 1 - a) \rangle$
$\vee_{1,130}$	$\langle 1 - \min(d, 1 - a), \min(d(1 - d) + \overline{\text{sg}}(1 - d), 1 - a) \rangle$
$\wedge_{1,130}$	$\langle \min(a, c), 1 - \min(c, a) \rangle$
$\vee_{1,131}$	$\langle \max(1 - d, (1 - (a(1 - a) + \overline{\text{sg}}(a)))(a(1 - a) + \overline{\text{sg}}(a)) + \overline{\text{sg}}((a(1 - a) + \overline{\text{sg}}(a))))), \min(d(1 - d) + \overline{\text{sg}}(1 - d), (a(1 - a) + \overline{\text{sg}}(a))((1 - (a(1 - a) + \overline{\text{sg}}(a)))(a(1 - a) + \overline{\text{sg}}(a)) + \overline{\text{sg}}((a(1 - a) + \overline{\text{sg}}(a)))) + \overline{\text{sg}}(1 - (a(1 - a) + \overline{\text{sg}}(a)))) \rangle$
$\wedge_{1,131}$	$\langle \max(0, (((1 - \max((1 - \min(1, ((c(((1 - c)c) + \overline{\text{sg}}(c))) + \overline{\text{sg}}(1 - c))))), (((1 - a)a) + \overline{\text{sg}}(a)))) \max((1 - \min(1, ((c(((1 - c)c) + \overline{\text{sg}}(c))) + \overline{\text{sg}}(1 - c))))), (((1 - a)a) + \overline{\text{sg}}(a)))) + \overline{\text{sg}}(\max((1 - \min(1, ((c(((1 - c)c) + \overline{\text{sg}}(c))) + \overline{\text{sg}}(1 - c))))), (((1 - a)a) + \overline{\text{sg}}(a))))), \min(1, ((\max((1 - \min(1, ((c(((1 - c)c) + \overline{\text{sg}}(c))) + \overline{\text{sg}}(1 - c))))), (((1 - a)a) + \overline{\text{sg}}(a))))), \min(1, ((\max((1 - \min(1, ((c(((1 - c)c) + \overline{\text{sg}}(c))) + \overline{\text{sg}}(1 - c))))), (((1 - a)a) + \overline{\text{sg}}(a)))))) \rangle$

	$((1-a)a + \overline{sg}(a))((1 - \max((1 - \min(1, ((c((1-c)c) + \overline{sg}(c)) + \overline{sg}(1-c))))), ((1-a)a + \overline{sg}(a)))) \max((1 - \min(1, ((c((1-c)c) + \overline{sg}(c)) + \overline{sg}(1-c))))), ((1-a)a + \overline{sg}(a))) + \overline{sg}(\max((1 - \min(1, ((c((1-c)c) + \overline{sg}(c)) + \overline{sg}(1-c))))), ((1-a)a + \overline{sg}(a)))) + \overline{sg}(1 - \max((1 - \min(1, ((c((1-c)c) + \overline{sg}(c)) + \overline{sg}(1-c))))), ((1-a)a + \overline{sg}(a))))))$
$\vee_{1,132}$	$\langle 1 - (1-a)d, (d(1-d) + \overline{sg}(1-d))(1-a) \rangle$
$\wedge_{1,132}$	$\langle ac, 1-ac \rangle$
$\vee_{1,133}$	$\langle (1-d+a(1-a))(1-a(1-a)) - (1-d)(a(1-a)) \cdot (1-a(1-a) + \overline{sg}(a(1-a))), (d(1-d) + \overline{sg}(1-d)) \cdot ((1-a)a(a(1-a)(1-a(1-a)) + \overline{sg}(a(1-a))) + \overline{sg}(1-a(1-a))) \rangle$
$\wedge_{1,133}$	$\langle (-c(c(1-c) + \overline{sg}(c)) + \overline{sg}(1-c)) + a(1-a)(1-a(1-a)) + (c(c(1-c) + \overline{sg}(c)) - \overline{sg}(1-c))(a(1-a)(1-a(1-a)) + \overline{sg}(a(1-a)(1-a(1-a))))(1-c(c(1-c) + \overline{sg}(c)) + \overline{sg}(1-c)) + a(1-a)(1-a(1-a)) + (c(c(1-c) + \overline{sg}(c)) - \overline{sg}(1-c))(a(1-a)(1-a(1-a)) + \overline{sg}(a(1-a)(1-a(1-a))))), -c(c(1-c) + \overline{sg}(c)) + \overline{sg}(1-c) + a(1-a)(1-a(1-a)) + (c(c(1-c) + \overline{sg}(c)) - \overline{sg}(1-c))(a(1-a)(1-a(1-a)) + \overline{sg}(a(1-a)(1-a(1-a)))) \cdot (1-a(1-a))) \rangle$
$\vee_{1,134}$	$\langle c + \min(\overline{sg}(c), 1-b), (1-c)c + \min(\overline{sg}(c), b) \rangle$
$\wedge_{1,134}$	$\langle (1-d)d + \min(\overline{sg}(d), 1-b), 1 - (1-d)d - \min(\overline{sg}(d), 1-b) \rangle$
$\vee_{1,135}$	$\langle \max(1-b, c), \min((1-c)c + \overline{sg}(c), b) \rangle$
$\wedge_{1,135}$	$\langle \min((1-d)d + \overline{sg}(d), 1-b), 1 - \min((1-d)d + \overline{sg}(d), 1-b) \rangle$
$\vee_{1,136}$	$\langle \max(c, (1-b)(b(1-b) + \overline{sg}(1-b))(1 - (1-b)(b(1-b) + \overline{sg}(1-b)))) + \overline{sg}(1-b)), \min((1-c)c + \overline{sg}(c), (1 - (1-b)(b(1-b) + \overline{sg}(1-b))) \cdot ((1-b)(b(1-b) + \overline{sg}(1-b))(1 - (1-b)(b(1-b) + \overline{sg}(1-b))) + \overline{sg}(1-b))) + \overline{sg}(1 - (1-b)(b(1-b) + \overline{sg}(1-b)))) \rangle$
$\wedge_{1,136}$	$\langle \min((\overline{sg}(1-d) - d(1-d))(d(1-d) + \overline{sg}(1-d)) + \overline{sg}(d(1-d) + \overline{sg}(1-d))), (\overline{sg}(b) - (1-b)(b(1-b) + \overline{sg}(1-b)))((1-b)(b(1-b) + \overline{sg}(1-b)) + \overline{sg}(b))(\overline{sg}(b) - (1-b)(b(1-b) + \overline{sg}(1-b))) + \overline{sg}(\overline{sg}(b) - (1-b)(b(1-b) + \overline{sg}(1-b)))) + \overline{sg}((1-b)(b(1-b) + \overline{sg}(1-b)) + \overline{sg}(b))), 1 - \min((\overline{sg}(1-d) - d(1-d))(d(1-d) + \overline{sg}(1-d)) + \overline{sg}(d(1-d) + \overline{sg}(1-d))), (\overline{sg}(b) - (1-b)(b(1-b) + \overline{sg}(1-b)))((1-b)(b(1-b) + \overline{sg}(1-b)) + \overline{sg}(b))(\overline{sg}(b) - (1-b)(b(1-b) + \overline{sg}(1-b))) + \overline{sg}(\overline{sg}(b) - (1-b)(b(1-b) + \overline{sg}(1-b)))) \rangle$

$\wedge_{1,146}$	$\langle \frac{2}{3} - 2 \frac{8-3a-2c-\min(3a,1+2c)}{27}, \frac{1}{3} + 2 \frac{8-3a-2c-\min(3a,1+2c)}{27} \rangle$
$\vee_{1,147}$	$\langle \frac{1+2a+3c+\max(1+2a,3c)}{9}, 1 - \frac{1+2a+3c+\max(1+2a,3c)}{9} \rangle$
$\wedge_{1,147}$	$\langle \frac{2}{3} - 2 \frac{5-3a-2c+\max(3-3a,2-2c)}{27}, \frac{1}{3} + 2 \frac{5-3a-2c+\max(3-3a,2-2c)}{27} \rangle$
$\vee_{1,148}$	$\langle \frac{6-2b-3d+\max(3-2b,3-3d)}{9}, 1 - \frac{6-2b-3d+\max(3-2b,3-3d)}{9} \rangle$
$\wedge_{1,148}$	$\langle \frac{2}{3} - \frac{9-3b-2d-\max(3b,2d)}{9}, 1 - \frac{2}{3} - \frac{9-3b-2d-\max(3b,2d)}{9} \rangle$
$\vee_{1,149}$	$\langle \frac{3-2b+3c+\max(3-2b,3c)}{9}, 1 - \frac{3-2b+3c+\max(3-2b,3c)}{9} \rangle$
$\wedge_{1,149}$	$\langle \frac{2}{3} - \frac{9-3b-2d-\max(3b,2d)}{9}, 1 - \frac{2}{3} - \frac{9-3b-2d-\max(3b,2d)}{9} \rangle$
$\vee_{1,150}$	$\langle \frac{a+\lambda+(2\lambda)(c+\lambda-1)}{4\lambda^2}, \frac{b+\lambda-1+(2\lambda)(d+\lambda-1)}{4\lambda^2} \rangle$
$\wedge_{1,150}$	$\langle \frac{a+(2\lambda-1)\lambda+2\lambda(c+\lambda)+4\lambda^2(\lambda-1)-2\lambda}{16\lambda^4} + \frac{1}{2}, \frac{b+(2\lambda+1)(\lambda-1)+2\lambda(d+\lambda-1)+4\lambda^2(\lambda-1)}{8\lambda^3} + \frac{1}{2} \rangle$
$\vee_{1,151}$	$\langle \frac{a+\gamma+(2\gamma+1)(c+\gamma)}{(2\gamma+1)^2}, \frac{b+\gamma+(2\gamma+1)(d+\gamma-1)}{(2\gamma+1)^2} \rangle$
$\wedge_{1,151}$	$\langle \frac{\gamma}{2\gamma+1} + \frac{a+2\gamma(\gamma+1)+(c+\gamma)(2\gamma+1)+(2\gamma+1)^2(\gamma-1)}{(2\gamma+1)^4}, \frac{\gamma}{2\gamma+1} + \frac{b+2\gamma(\gamma+1)+(d+\gamma)(2\gamma+1)+\gamma(2\gamma+1)^2}{(2\gamma+1)^4} \rangle$
$\vee_{1,152}$	$\langle \frac{a+\beta+(\alpha+\beta)(c+\alpha-1)}{(\alpha+\beta)^2}, \frac{b+\alpha-1+(\alpha+\beta)(d+\beta-1)}{(\alpha+\beta)^2} \rangle$
$\wedge_{1,152}$	$\langle \frac{\alpha-1}{\alpha+\beta} + \frac{a+(\alpha-1)(\alpha+\beta)+\beta+(c+\beta)(\alpha+\beta)+(\beta-1)(\alpha+\beta)^2}{(\alpha+\beta)^4}, \frac{\beta}{\alpha+\beta} + \frac{b+\alpha-1+\beta(\alpha+\beta)+(d+\alpha-1)(\alpha+\beta)+(\alpha-1)(\alpha+\beta)^2}{(\alpha+\beta)^4} \rangle$
$\vee_{1,153}$	$\langle \min(1, \max(c, \max(0, a - \eta) + \varepsilon)), \max(0, \min(d, \min(1, b + \varepsilon) - \eta)) \rangle$
$\wedge_{1,153}$	$\langle \min(1, \max(0, \min(\max(0, c - \eta), \min(1, \max(0, a - \eta) + \varepsilon) - \eta)) + \varepsilon), \max(0, \min(1, \max(\min(1, d + \varepsilon), \max(0, \min(1, b + \varepsilon) - \eta) + \varepsilon)) - \eta) \rangle$
$\vee_{1,154}$	$\langle \frac{-\lambda+a+(2\lambda)(c+\lambda)}{4\lambda^2}, \frac{\lambda-a+(2\lambda)(\lambda-c)}{4\lambda^2} \rangle$
$\wedge_{1,154}$	$\langle \frac{(4\lambda^3-2\lambda^2+c)(2\lambda)-\lambda+a}{16\lambda^4}, \frac{(4\lambda^3+2\lambda^2-c)(2\lambda)+\lambda-a}{16\lambda^4} \rangle$
$\vee_{1,155}$	$\langle \frac{-\lambda+a+2\lambda(1-d+\lambda)}{4\lambda^2}, 1 - \frac{-\lambda+a+2\lambda(1-d+\lambda)}{4\lambda^2} \rangle$
$\wedge_{1,155}$	$\langle \frac{4\lambda^2(2\lambda^2-1-\lambda)+2\lambda(2\lambda+c)-\lambda+a}{16\lambda^4}, 1 - \frac{4\lambda^2(2\lambda^2-1-\lambda)+2\lambda(2\lambda+c)-\lambda+a}{16\lambda^4} \rangle$
$\vee_{1,156}$	$\langle \frac{1-b+\lambda+2\lambda(c+\lambda-1)}{4\lambda^2}, 1 - \frac{1-b+\lambda+2\lambda(c+\lambda-1)}{4\lambda^2} \rangle$
$\wedge_{1,156}$	$\langle \frac{8\lambda^4-4\lambda^3-2d\lambda+1-b+\lambda}{16\lambda^4}, 1 - \frac{8\lambda^4-4\lambda^3-2d\lambda+1-b+\lambda}{16\lambda^4} \rangle$
$\vee_{1,157}$	$\langle \frac{1-b+\lambda+2\lambda(-d+\lambda)}{4\lambda^2}, 1 - \frac{1-b+\lambda+2\lambda(-d+\lambda)}{4\lambda^2} \rangle$
$\wedge_{1,157}$	$\langle \frac{4\lambda^3-2\lambda^2+2d\lambda-1+b-\lambda}{4\lambda^2}, 1 - \frac{4\lambda^3-2\lambda^2+2d\lambda-1+b-\lambda}{4\lambda^2} \rangle$
$\vee_{1,158}$	$\langle \frac{-1+a-\gamma+(2\gamma+1)(1+c+\gamma)}{(2\gamma+1)^2}, 1 - \frac{-1+a-\gamma+(2\gamma+1)(1+c+\gamma)}{(2\gamma+1)^2} \rangle$
$\wedge_{1,158}$	$\langle 1 - \frac{(2\gamma+1)^3\gamma+(\gamma+1)(2\gamma+1)^2-c(2\gamma+1)+1-\alpha+\gamma}{(2\gamma+1)^4}, \frac{(2\gamma+1)^3\gamma+(\gamma+1)(2\gamma+1)^2-c(2\gamma+1)+1-\alpha+\gamma}{(2\gamma+1)^4} \rangle$

$\vee_{1,159}$	$\langle \frac{-1+a-\gamma+(2\gamma+1)(2-d+\gamma)}{(2\gamma+1)^2}, 1 - \frac{-1+a-\gamma+(2\gamma+1)(2-d+\gamma)}{(2\gamma+1)^2} \rangle$
$\wedge_{1,159}$	$\langle 1 - \frac{(2\gamma+1)^3\gamma+(\gamma+2)(2\gamma+1)^2-(2\gamma+1)(c+1+2\gamma)+1-a+\gamma}{(2\gamma+1)^4}, \frac{(2\gamma+1)^3\gamma+(\gamma+2)(2\gamma+1)^2-(2\gamma+1)(c+1+2\gamma)+1-a+\gamma}{(2\gamma+1)^4} \rangle$
$\vee_{1,160}$	$\langle \frac{1-b+\gamma+(2\gamma+1)(-d+\gamma+1)}{(2\gamma+1)^2}, 1 - \frac{1-b+\gamma+(2\gamma+1)(-d+\gamma+1)}{(2\gamma+1)^2} \rangle$
$\wedge_{1,160}$	$\langle \frac{(2\gamma+1)^3\gamma+\gamma(2\gamma+1)^2-d(2\gamma+1)+1-b+\gamma}{(2\gamma+1)^4}, 1 - \frac{(2\gamma+1)^3\gamma+\gamma(2\gamma+1)^2-d(2\gamma+1)+1-b+\gamma}{(2\gamma+1)^4} \rangle$
$\vee_{1,161}$	$\langle \frac{1-b+\gamma+(2\gamma+1)(c+\gamma)}{(2\gamma+1)^2}, 1 - \frac{1-b+\gamma+(2\gamma+1)(c+\gamma)}{(2\gamma+1)^2} \rangle$
$\wedge_{1,161}$	$\langle \frac{(2\gamma+1)^3\gamma+(\gamma+1)(2\gamma+1)^2-(2\gamma+1)(d+1+\gamma)+1-b+\gamma}{(2\gamma+1)^4}, 1 - \frac{(2\gamma+1)^3\gamma+(\gamma+1)(2\gamma+1)^2-(2\gamma+1)(d+1+\gamma)+1-b+\gamma}{(2\gamma+1)^4} \rangle$
$\vee_{1,162}$	$\langle \frac{-b+1-\alpha+(\alpha+\beta)(c+\alpha)}{(\alpha+\beta)^2}, 1 - \frac{-b+1-\alpha+(\alpha+\beta)(c+\alpha)}{(\alpha+\beta)^2} \rangle$
$\wedge_{1,162}$	$\langle 1 - \frac{\beta(\alpha+\beta)^3+\beta(\alpha+\beta)^2-d(\alpha+\beta)+a+\beta}{(\alpha+\beta)^4}, \frac{\beta(\alpha+\beta)^3+\beta(\alpha+\beta)^2-d(\alpha+\beta)+a+\beta}{(\alpha+\beta)^4} \rangle$
$\vee_{1,163}$	$\langle \frac{2-b-\alpha+(\alpha+\beta)(-d+\alpha)}{(\alpha+\beta)^2}, 1 - \frac{2-b-\alpha+(\alpha+\beta)(-d+\alpha)}{(\alpha+\beta)^2} \rangle$
$\wedge_{1,163}$	$\langle 1 - \frac{\beta(\alpha+\beta)^3+\beta(\alpha+\beta)^2+(c-1)(\alpha+\beta)+a+\beta}{(\alpha+\beta)^4}, \frac{\beta(\alpha+\beta)^3+\beta(\alpha+\beta)^2+(c-1)(\alpha+\beta)+a+\beta}{(\alpha+\beta)^4} \rangle$
$\vee_{1,164}$	$\langle \frac{a+\beta+(\alpha+\beta)(-d+\alpha)}{(\alpha+\beta)^2}, 1 - \frac{a+\beta+(\alpha+\beta)(-d+\alpha)}{(\alpha+\beta)^2} \rangle$
$\wedge_{1,164}$	$\langle \frac{(\alpha-1)(\alpha+\beta)^2+(\alpha+\beta)(b+\alpha)-c-\beta}{(\alpha+\beta)^3}, 1 - \frac{(\alpha-1)(\alpha+\beta)^2+(\alpha+\beta)(b+\alpha)-c-\beta}{(\alpha+\beta)^3} \rangle$
$\vee_{1,165}$	$\langle \frac{a+\beta+(\alpha+\beta)(c+\alpha-1)}{(\alpha+\beta)^2}, 1 - \frac{a+\beta+(\alpha+\beta)(c+\alpha-1)}{(\alpha+\beta)^2} \rangle$
$\wedge_{1,165}$	$\langle \frac{(\alpha-1)(\alpha+\beta)^2+(\alpha+\beta)(1-b)-d+1-\alpha}{(\alpha+\beta)^3}, 1 - \frac{(\alpha-1)(\alpha+\beta)^2+(\alpha+\beta)(1-b)-d+1-\alpha}{(\alpha+\beta)^3} \rangle$
$\vee_{1,166}$	$\langle \max(a, \min(b, c)), \min(b, \max(a, d)) \rangle$
$\wedge_{1,166}$	$\langle \min(a, \max(b, c)), \max(b, \min(a, d)) \rangle$
$\vee_{1,167}$	$\langle \max(1-a, \min(1-a, c)), \min(1-a, 1-\min(1-a, c)) \rangle$
$\wedge_{1,167}$	$\langle 1-\max(a, \min(a, 1-c)), \max(a, \min(a, 1-c)) \rangle$
$\vee_{1,168}$	$\langle \max(a, \min(1-a, 1-d)), 1-\max(a, \min(1-a, 1-d)) \rangle$
$\wedge_{1,168}$	$\langle 1-\max(1-a, \min(a, c)), \max(1-a, \min(a, c)) \rangle$
$\vee_{1,169}$	$\langle \max(1-b, \min(b, c)), 1-\max(1-b, \min(b, c)) \rangle$
$\wedge_{1,169}$	$\langle 1-\max(b, \min(1-b, d)), \max(b, \min(1-b, d)) \rangle$
$\vee_{1,170}$	$\langle \max(1-b, \min(b, 1-d)), 1-\max(1-b, \min(b, 1-d)) \rangle$
$\wedge_{1,170}$	$\langle 1-\max(b, \min(1-b, d)), \max(b, \min(1-b, d)) \rangle$
$\vee_{1,171}$	$\langle \overline{\text{sg}}(\max(\overline{\text{sg}}(1-b), d) - \max(\text{sg}(1-b), c)), \text{sg}(\max(\overline{\text{sg}}(1-b), d) - \max(\text{sg}(1-b), c))) \rangle$
$\wedge_{1,171}$	$\langle \text{sg}(\max(a, \text{sg}(1-d)) - \max(b, \overline{\text{sg}}(1-d))), \overline{\text{sg}}(\max(a, \text{sg}(1-d)) - \max(b, \overline{\text{sg}}(1-d))) \rangle$
$\vee_{1,172}$	$\langle \overline{\text{sg}}(\overline{\text{sg}}(a) - c), \text{sg}(\overline{\text{sg}}(a) - c) \rangle$
$\wedge_{1,172}$	$\langle \text{sg}(a - \overline{\text{sg}}(c)), \overline{\text{sg}}(a - \overline{\text{sg}}(c)) \rangle$
$\vee_{1,173}$	$\langle \overline{\text{sg}}(-\text{sg}(a) + d), \text{sg}(-\text{sg}(a) + d) \rangle$
$\wedge_{1,173}$	$\langle \overline{\text{sg}}(a - \overline{\text{sg}}(c)), \text{sg}(a - \overline{\text{sg}}(c)) \rangle$

$\vee_{1,174}$	$\langle \overline{\text{sg}}(\text{sg}(1-b) - c), \text{sg}(\text{sg}(1-b) - c) \rangle$
$\wedge_{1,174}$	$\langle \text{sg}(1-b - \overline{\text{sg}}(1-d)), \overline{\text{sg}}(1-b - \overline{\text{sg}}(1-d)) \rangle$
$\vee_{1,175}$	$\langle \text{sg}(\text{sg}(1-d) - b), \overline{\text{sg}}(\text{sg}(1-d) - b) \rangle$
$\wedge_{1,175}$	$\langle \text{sg}(\text{sg}(1-d) - \overline{\text{sg}}(1-b)), \overline{\text{sg}}(\text{sg}(1-d) - \overline{\text{sg}}(1-b)) \rangle$
$\vee_{1,176}$	$\langle \overline{\text{sg}}(\overline{\text{sg}}(a) + \text{sg}(a)b - c) + \text{sg}(\overline{\text{sg}}(a) + \text{sg}(a)b - c) \max(a, c),$ $\text{sg}(\overline{\text{sg}}(a) + \text{sg}(a)b - c) \min(\overline{\text{sg}}(a) + \text{sg}(a)b, d) \rangle$
$\wedge_{1,176}$	$\langle \text{sg}(\max(a, \text{sg}(1-d)) - \max(b, \overline{\text{sg}}(1-d))),$ $\overline{\text{sg}}(\max(a, \text{sg}(1-d)) - \max(b, \overline{\text{sg}}(1-d))) \rangle$
$\vee_{1,177}$	$\langle \overline{\text{sg}}(\overline{\text{sg}}(a) + \text{sg}(a)(1-a) - c) + \text{sg}(\overline{\text{sg}}(a) + \text{sg}(a)(1-a) - c) \max(\text{sg}(a)(2-a), c),$ $\text{sg}(\overline{\text{sg}}(a) + \text{sg}(a)(1-a) - c) \cdot \min(\overline{\text{sg}}(a) + \text{sg}(a)(1-a), 1-c) \rangle$
$\wedge_{1,177}$	$\langle \text{sg}(a - \overline{\text{sg}}(c)), \overline{\text{sg}}(a - \overline{\text{sg}}(c)) \rangle$
$\vee_{1,178}$	$\langle \overline{\text{sg}}(-a\text{sg}(a) + d) + \text{sg}(a - 1 + d)(1 - \min(\overline{\text{sg}}(a) + \text{sg}(a)(1-a), d)),$ $\text{sg}(-a\text{sg}(a) + d) \min(\overline{\text{sg}}(a) + \text{sg}(a)(1-a), d) \rangle$
$\wedge_{1,178}$	$\langle \text{sg}((a - \overline{\text{sg}}(c))), \overline{\text{sg}}((a - \overline{\text{sg}}(c))) \rangle$
$\vee_{1,179}$	$\langle \overline{\text{sg}}(a - c) + \text{sg}(a - c) \max(1 - a, c),$ $\text{sg}(a - c)(1 - \max(1 - a, c)) \rangle$
$\wedge_{1,179}$	$\langle \text{sg}(\text{sg}(1-d) - b), \overline{\text{sg}}(\text{sg}(1-d) - b) \rangle$
$\vee_{1,180}$	$\langle \overline{\text{sg}}(d - 1 + a) + \text{sg}(d - 1 + a) \max(1 - a, 1 - d),$ $\text{sg}(d - 1 + a) \min(a, d) \rangle$
$\wedge_{1,180}$	$\langle \overline{\text{sg}}(\text{sg}(-c + \overline{\text{sg}}(b) + b\text{sg}(1-b)) \min(\overline{\text{sg}}(b) + b\text{sg}(1-b), 1-c)) + \text{sg}(1 - \text{sg}(-c + \overline{\text{sg}}(b) + b\text{sg}(1-b)) \min(\overline{\text{sg}}(b) + b\text{sg}(1-b), 1-c))$ $(\text{sg}(-c + \overline{\text{sg}}(b) + b\text{sg}(1-b)) \min(\overline{\text{sg}}(b) + b\text{sg}(1-b), 1-c)),$ $\text{sg}(\overline{\text{sg}}(b) - c + b\text{sg}(1-b)) - \text{sg}(-c + \overline{\text{sg}}(b) + b\text{sg}(1-b)) \max(\text{sg}(b) - b\text{sg}(1-b), c) \rangle$
$\vee_{1,181}$	$\langle 1 - \overline{\text{sg}}(a) \cdot (1 - c), d \cdot \overline{\text{sg}}(a) \rangle$
$\wedge_{1,181}$	$\langle \text{sg}(\text{sg}(a) \cdot \text{sg}(c)), \overline{\text{sg}}(\text{sg}(a) \cdot \text{sg}(c)) \rangle$
$\vee_{1,182}$	$\langle 1 - \overline{\text{sg}}(a) \cdot (1 - c), (1 - c) \cdot \overline{\text{sg}}(a) \rangle$
$\wedge_{1,182}$	$\langle \text{sg}(\text{sg}(a) \cdot \text{sg}(c)), \overline{\text{sg}}(\text{sg}(a) \cdot \text{sg}(c)) \rangle$
$\vee_{1,183}$	$\langle 1 - \overline{\text{sg}}(a) \cdot d, d \cdot \overline{\text{sg}}(a) \rangle$
$\wedge_{1,183}$	$\langle \text{sg}(\text{sg}(a) \cdot \text{sg}(c)), \overline{\text{sg}}(\text{sg}(a) \cdot \text{sg}(c)) \rangle$
$\vee_{1,184}$	$\langle 1 - \overline{\text{sg}}(1-b) \cdot d, d \cdot \overline{\text{sg}}(1-b) \rangle$
$\wedge_{1,184}$	$\langle \overline{\text{sg}}(1 - \text{sg}(1-d) \cdot \text{sg}(1-b)), \text{sg}(1 - \text{sg}(1-d) \cdot \text{sg}(1-b)) \rangle$
$\vee_{1,185}$	$\langle 1 - \overline{\text{sg}}(1-b) \cdot (1 - c), (1 - c) \cdot \overline{\text{sg}}(1-b) \rangle$
$\wedge_{1,185}$	$\langle \overline{\text{sg}}(1 - \text{sg}(1-d) \cdot \text{sg}(1-b)), \text{sg}(1 - \text{sg}(1-d) \cdot \text{sg}(1-b)) \rangle$

3 Conclusion

With the constructed conjunctions and disjunctions we will consider algebraic structures such as monoids, groups and lattices and study their properties.

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