GENERALIZED NETS HAVING PLACES WITH LIMITED GLOBAL CAPACITIES

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A variety of different types of Generalized Net (GN) extensions have been defined and each of them has been proven to be a conservative extension of the ordinary GNs (see e.g. [1,2]).

In this note, we introduce yet another extension and prove that it is a conservative one.

We use the following notations throughout:

- \( \mathcal{N} = \{0, 1, 2, \ldots \} \cup \{\infty\} \);
- \( pr_i X \) is the \( i \)-th projection of the \( n \)-dimensional set, where \( n \in \mathcal{N}, n \geq 1 \), and \( 1 \leq i \leq n \). More generally, for a given \( n \)-dimensional set \( X \) \( (n \geq 2) \)

\[
pr_{i_1, i_2, \ldots, i_k} X = \prod_{j=1}^{k} pr_{i_j} X
\]

where \( 1 \leq i_1 < i_2 < \ldots < i_k \leq n \).
- \( card(X) \) is the cardinality of set \( X \).

As for the GN-specific notation, we refer the reader to [1,2].

The formal definition of the new type of extension coincide with this of the ordinary GN. Every transition is described by a seven-tuple (see Fig. 1):

\[
Z = (L', L'', t_1, t_2, r, M, \square),
\]

where:
(a) $L'$ and $L''$ are finite, non-empty sets of places (the transition's input and output places, respectively); for the transition in Fig. 1 these are

$$L' = \{ l'_1, l'_2, \ldots, l'_m \}$$

and

$$L'' = \{ l''_1, l''_2, \ldots, l''_n \};$$

(b) $t_1$ is the current time-moment of the transition’s firing;

(c) $t_2$ is the current value of the duration of its active state;

(d) $r$ is the transition’s condition determining which tokens will transfer from the transition’s inputs to its outputs. Parameter $r$ has the form of an Index Matrix (IM, see [1,2]):

$$r = \begin{bmatrix}
    l'_1 & l''_1 & \ldots & l''_n \\
    \vdots & \vdots & \ddots & \vdots \\
    l'_i & r_{i,j} & & (1 \leq i \leq m, 1 \leq j \leq n) \\
    \vdots & & & \\
    l'_m & & & (1 \leq i \leq m, 1 \leq j \leq n)
\end{bmatrix}$$

where $r_{i,j}$ is the predicate which expresses the condition for transfer from the $i$-th input place to the $j$-th output place with the obvious meaning: whenever $r_{i,j}$ has truth-value “true”, a token from the $i$-th input place can be transferred to the $j$-th output place; otherwise, such a transfer is not allowed;

(e) $M$ is an IM of the capacities of transition’s arcs:

$$M = \begin{bmatrix}
    l'_1 & l''_1 & \ldots & l''_n \\
    \vdots & \vdots & \ddots & \vdots \\
    l'_i & m_{i,j} & & (1 \leq i \leq m, 1 \leq j \leq n) \\
    \vdots & & & \\
    l'_m & & & (1 \leq i \leq m, 1 \leq j \leq n)
\end{bmatrix}$$

(f) $\square$ is called transition type and it is an object having a form similar to a Boolean expression. It may contain as variables the symbols that serve as labels for transition’s input places, and it is an expression constructed of variables and the Boolean connectives $\land$ and $\lor$ determining the following conditions:

$$\land(l_{i_1}, l_{i_2}, \ldots, l_{i_u})$$

— every place $l_{i_1}, l_{i_2}, \ldots, l_{i_u}$ must contain at least one token,

$$\lor(l_{i_1}, l_{i_2}, \ldots, l_{i_u})$$

— there must be at least one token in the set of places $l_{i_1}, l_{i_2}, \ldots, l_{i_u}$, where \{\ i_1, i_2, \ldots, i_u \ \} \subset L'.$

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Whenever the value of a type (calculated as a Boolean expression) is “true”, the transition can become active, otherwise it is inactive.

The ordered four-tuple

\[ E = \langle \langle A, \pi_A, \pi_L, c, C, f, \theta_1, \theta_2 \rangle, \langle K, \pi_K, \theta_K \rangle, \langle T, r, t' \rangle, \langle X, \Phi, b \rangle \rangle \]

is called a Generalized Net having Places with Limited Capacities (GN-PLC) if:

(a) \( A \) is a set of transitions (see above);
(b) \( \pi_A \) is a function giving the priorities of the transitions, i.e., \( \pi_A : A \rightarrow \mathcal{N} \);
(c) \( \pi_L \) is a function giving the priorities of the places, i.e., \( \pi_L : L \rightarrow \mathcal{N} \), where

\[ L = pr_1A \cup pr_2A \]

and obviously, \( L \) is the set of all GN-places;

(d) \( c \) is a function giving the capacities of the places, i.e., \( c : L \rightarrow \mathcal{N} \);
(e) \( C \) is a function giving the global capacities of the places, i.e., \( c : L \rightarrow \mathcal{N} \). Here, the expression “global capacities” denotes that for a given place \( l \in \mathcal{L} \), \( C(l) \) is the maximal number of tokens that can enter place \( l \) during the functioning of the GN;

(f) \( f \) is a function that calculates the truth values of the predicates of the transition’s conditions; for the (ordinary) GNs, described in this section, function \( f \) obtain values “false” or “true”, or values from set \( \{0,1\} \); if \( P \) is the set of the predicates used in a given model, then we can define \( f \) as \( f : P \rightarrow \{0,1\} \);

(g) \( \theta_1 \) is a function giving the next-time moment, for which a given transition \( Z \) can be activated, i.e., \( \theta_1(t) = t' \), where \( pr_3Z = t, t' \in [T, T + t'] \) and \( t \leq t' \); the value of this function is calculated at the moment when the transition terminates its functioning;

(h) \( \theta_2 \) is a function giving the duration of the active state of a given transition \( Z \), i.e., \( \theta_2(t) = t' \), where \( pr_4Z = t \in [T, T + t'] \) and \( t' \geq 0 \); the value of this function is calculated at the moment when the transition starts functioning;

(i) \( K \) is the set of the GN’s tokens. In some cases, it is convenient to consider this set in the form

\[ K = \bigcup_{l \in Q_l} K_l, \]

where \( K_l \) is the set of tokens which enter the net from place \( l \), and \( Q_l \) is the set of all input places of the net.
(j) $\pi_K$ is a function giving the priorities of the tokens, i.e., $\pi_K : K \rightarrow \mathcal{N}$;
(k) $\theta_K$ is a function giving the time-moment when a given token can enter the net, i.e., $\theta_K(\alpha) = t$, where $\alpha \in K$ and $t \in [T, T + t^*]$;
(l) $T$ is the time-moment when the GN starts functioning; this moment is determined with respect to a fixed (global) time-scale;
(m) $t^0$ is an elementary time-step, related to the fixed (global) time-scale;
(n) $t^*$ is the duration of the GN functioning;
(o) in all publications on GNs (see, e.g., [1]) it is defined that $X$ is the set of all initial characteristics that the tokens can receive when they enter the net; in [2], for a first time another interpretation of $X$ will be introduced: $X$ is a function which assigns initial characteristics to every token when it enters input place of the net; if $\alpha \in K$, then it enters the GN with initial characteristic $x_{o}^{\alpha}$;
(p) $\Phi$ is a characteristic function that assigns new characteristics to every token when it makes a transfer from an input to an output place of a given transition; if $\alpha \in K$, then it, entering an output place of some GN-transition and having as current characteristic $x_{cu}^{\alpha}$, obtains the next characteristic $x_{cu+1}^{\alpha}$;
(q) $b$ is a function giving the maximum number of characteristics a given token can receive, i.e., $b : K \rightarrow \mathcal{N}$.

When for each place $l$ of a given GN: $c(l) = \infty$, then the GN-PLC is an ordinary GN. Therefore, a GN-PLC is an extension of a GN.

**Theorem:** The functioning and the results of the work of each GN-PLC can be represented by an ordinary GN.

**Proof.** Let $E$ be a given GN-PLC. We construct the ordinary GN $F$ with the form

$$F = \langle (A^*, \pi_A, \pi_L, c, f, \theta_1, \theta_2), (K^*, \pi_K, \theta_K), (T, t^0, t^*), (X^*, \Phi^*, b) \rangle,$$

where $A^*$ is the set of the $F$-transitions. Let transition $Z^*$ of $F$, corresponding to transition $Z$ of $E$, have the form

$$Z^* = \langle L''^*, L''*, t_1, t_2, r^*, M^*, \square \rangle,$$

(see Fig. 2) where $t_1$ and $t_2$ are as above and

$$L''^* = L' \cup \{l_Z\},$$

$$L''* = L'' \cup \{l_Z\},$$

$$\square^* = \wedge(\square, l_Z).$$

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If
\[ r = \text{pr}_5 Z = [L', L'', \{r_{t_i, t_j}\}] \]
has the form of an IM, then
\[ r^* = \text{pr}_5 Z^* = [L' \cup \{l_Z\}, L'' \cup \{l_Z\}, \{r_{t_i, t_j}\}], \]
where
\[ (\forall l_i \in L')(\forall l_j \in L'')(r_{t_i, t_j}^* = r_{t_i, t_j} \land \text{pr}_2[x_{t_i}^{\alpha_z}]_{t_j} > \text{pr}_2[x_{t_i}^{\alpha_z}]_{t_j}''), \]
where \( \text{pr}_2[x_{t_i}^{\alpha_z}]_{t_j} \) denotes the second component in couple \( lamgl_l_j, C(l_j) \) in \( k \)-th characteristic of token \( \alpha_z \) and \( x_{t_i}^{\alpha_z} \) is the current characteristic of token \( \beta \),

\[ (\forall l_i \in L')(\forall l_j \in L'')(r_{t_i, z}^* = r_{t_i, z} = \text{"false"}), \]
\[ r_{t_i, z} = \text{"true"}. \]

If
\[ M = \text{pr}_6 Z = [L', L'', \{m_{t_i, t_j}\}] \]
has the form of an IM, then
\[ M^* = \text{pr}_6 Z^* = [L' \cup \{l_Z\}, L'' \cup \{l_Z\}, \{m_{t_i, t_j}\}], \]
where
\[ (\forall l_i \in L')(\forall l_j \in L'')(m_{t_i, t_j}^* = m_{t_i, t_j}), \]
\[ (\forall l_i \in L')(\forall l_j \in L'')(m_{t_i, z}^* = m_{t_i, z} = 0), \]
\[ m_{t_i, z} = 1. \]

\[ \pi^*_L = \pi_L \cup \pi_{\{l_Z|Z \in A\}}, \]
where function \( \pi_{\{l_Z|Z \in A\}} \) determines the priorities of the new places, that are elements of set \( \{l_Z|Z \in A\} \) and the priorities of \( l_Z \)-places for every transition \( Z \in A \) are the minimum among the place priorities of this transition \( Z \).
\[ c'_L = c \cup c_{\{l_Z|Z \in A\}}, \]

where function \( c_{\{l_Z|Z \in A\}} \) satisfy equality

\[ c_{\{l_Z|Z \in A\}}(l_Z) = 1 \]

for all place \( l_Z \).

We shall note that if \( Q' \) is the set of the input places of the GN, then, as it is noted in [1,2] set \( K \) has representation

\[ K = \bigcup_{l \in Q'} K_l, \]

where \( K_l \) is the set of the GN-tokens that must enter the GN through place \( l \). Let us assume that these sets are ordered according the time-moments in which tokens will enter the GN. Let \( K^* \subset K_l \) is the set of the first \( C(l) \) tokens that will enter the GN.

\[ K^* = (\bigcup_{l \in Q'} K^*_l) \cup \{ \alpha_Z|Z \in A\}, \]

\[ \theta^*_K = \theta_K \cup \theta_{\{l_Z|Z \in A\}}, \]

where function \( \theta_{\{l_Z|Z \in A\}} \) determines that each \( \alpha_Z \)-token will stays in the initial time-moment \( T \) in its place.

\[ X^* = X \cup \{ x_0^{\alpha_Z}|Z \in A\}, \]

where \( x_0^{\alpha_Z} \) is the initial \( \alpha_Z \)-token characteristic and it is

\[ "\{(l_j,C(l_j))|l_j \in L''\}". \]

\[ \Phi^* = \Psi \cup \Psi_{\{l_Z|Z \in A\}}, \]

where function \( \Psi_{\{l_Z|Z \in A\}} \) determines the characteristics of the \( \alpha_Z \)-tokens in the form

\[ \Psi_{\{l_Z|Z \in A\};(\alpha_Z} = "\{(l_j, \lambda(l_j))|l_j \in L''\}". \]

where \( \lambda(l) \) is the number of tokens entering place \( l \).

We can now prove that GNSs \( E \) and \( F' \) are equivalent. To this end, we shall compare the functioning of a transition \( Z \) of GN \( E \) and its respective transition \( Z^* \) from \( F \). Obviously, these transitions start and stop working at precisely the same times. They also have equal priorities, the respective places of the transitions have equal capacities and priorities, and the arcs have equal capacities. Let us see the process of transferring tokens at a place \( l \) of \( Z \) and its respective place \( l^* \) in \( Z^* \). We need to consider two cases: the case of an input place and the case of an output place. In the former case, the place can be an input for the entire GN or not. If the place is an input for the transition, but not for the net, it clearly will be an output for another transition and, thus, its behaviour can be discussed as it were an output place, so let us concentrate on the case in which the places \( l \) and \( l^* \) are inputs for the entire GN. The place \( l \) will admit at most \( C(l) \) tokens (since this is its global capacity). On the other hand, the
place $l'$ admits the same number of tokens at, simply because by construction this is the capacity of set $K_l$. The tokens, which enter these two places, come with equal characteristics, because of the forms of the characteristic functions of both ncts. Now, for the second case, in which both places are outputs for their respective transitions, we can see that if a token can enter place $l$ (because the current number of tokens having entered it is smaller than its capacity), its counterpart in GN $l'$ can enter place $l'$, because the corresponding predicate has the same truth-values as in the first net. Also, both tokens obtain equal characteristics. Therefore, both places will have equal behaviour, and this will be valid for all $Z$ and respective $Z'$ places (without place $l_Z$). Hence, the two transitions have precisely the same behaviour. Thus proves the theorem.

In the future, we intend to discuss the optimal way of token entering the places with global capacities. If each token can enter such place, it is possible, its resource to receive new tokens will finish very quickly and in some cases this can generate difficulties of the GN-functioning. Therefore, it will be important the construct special algorithms determining which tokens to enter places with global capacities.

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References
